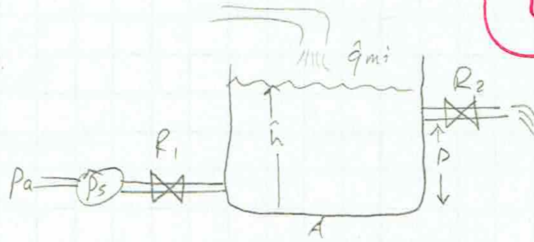
100

1 For the water tank system in #7.20, assume linear resistances + do the following:



a)

Inputs: $P_s, \hat{q}_{mi}, D$

Outputs: h

Parameters: $\rho, A, g, R_1, R_2,$

b) Write the corresponding diff EQs.

stored + out = in

$$C\frac{dp}{dt} = \frac{dm}{dt} = \rho A \frac{dh}{dt} = q_{mi} - q_{mo}$$

$$\rho A \frac{dh}{dt} = \hat{q}_{mi} + \frac{P_s - \rho g \hat{h}}{R_1} - \frac{(h-D)\rho g}{R_2}$$

$$\boxed{\rho A \frac{dh}{dt} + \rho g\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h = \hat{q}_{mi} + \frac{P_s}{R_1} + \frac{\rho g D}{R_2}}$$

$\frac{Kg}{s}$ ρ $\frac{m^3}{s}$ $\frac{m^3}{Kg}$

c) Obtain the time constant.

$$\boxed{\tau = \frac{A}{g(1/R_1 + 1/R_2)}}$$

d) Build the state-space model. Indicate A, B, C, D if tank Volume + volumetric flow accumulated.

$$\left[\frac{dh}{dt}\right] = \underbrace{\left[\frac{-g\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}{A}\right]}_{A}[h] + \underbrace{\begin{bmatrix} \frac{1}{\rho A} & \frac{1}{R_1 \rho A} & \frac{g}{R_2 A} \end{bmatrix}}_{B} \begin{bmatrix} \hat{q}_{mi} \\ P_s \\ D \end{bmatrix}$$

$\boxed{C}$ $\boxed{D}$

$$\begin{bmatrix} V \\ A\frac{dh}{dt} \end{bmatrix} = \begin{bmatrix} A \\ -g\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \end{bmatrix} [h] + \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\rho} & \frac{1}{R_1 \rho} & \frac{g}{R_2} \end{bmatrix} \begin{bmatrix} \hat{q}_{mi} \\ P_s \\ D \end{bmatrix}$$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$V = A \cdot h$

$$\text{vol. flow acc.} = \hat{q}_{mi} + \frac{P_s - \rho g \hat{h}}{R_1} - \frac{(h-D)\rho g}{R_2}$$

$$\rho A \frac{dh}{dt} = q_{mi} + \frac{P_s}{R_1} - \frac{\rho g h}{R_1} - \frac{\rho g h}{R_2} + \frac{\rho g D}{R_2}$$

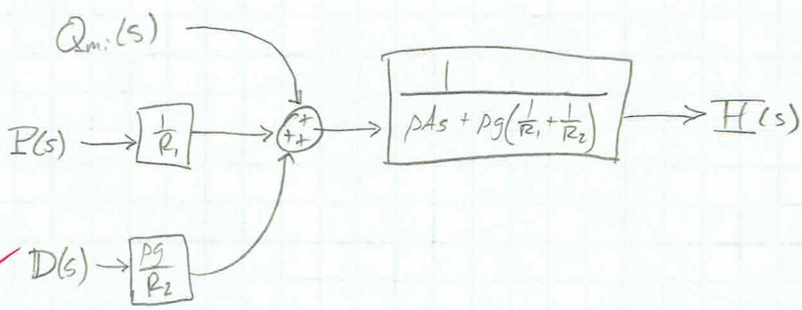
$$\rho A \frac{dh}{dt} = q_{mi} +$$

$\rho = \frac{m}{V}$

e) Find the corresponding TFs.

$$H(s)\left(\rho A s + \rho g\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right) = Q_{mi}(s) + \frac{P(s)}{R_1} + \frac{\rho g D(s)}{R_2}$$

$\frac{H(s)}{Q_{mi}(s)} = \frac{1}{\rho A s + \rho g\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$
$\frac{H(s)}{P(s)} = \frac{1/R_1}{\rho A s + \rho g(1/R_1 + 1/R_2)}$
$\frac{H(s)}{D(s)} = \frac{\rho g / R_2}{\rho A s + \rho g\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$



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f) IF $A = 2m^2$, $p_s = 80\,kPa$, $R_1 = 50\ m^{-1}s^{-1}$, $R_2 = 100\ m^{-1}s^{-1}$, $q_{mi} = 2\ \frac{kg}{s}$, $D = 3m$

Find the τ. $\left(= \frac{A}{g\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\right) = \frac{2}{(9.81)\left(\frac{1}{50}+\frac{1}{100}\right)} = 6.7958$

$Q(s) = 2/s$
$D(s) = 3/s$
$P_s(s) = 80000/s$
$\rho = 1000$

SS height $\longrightarrow$ $H(s) = \dfrac{\frac{2}{s} + \frac{80\,000}{50\,s} + \frac{(\cancel{1000})(9.81)\,3}{\cancel{100}\ s}}{(1000)(2)s + (1000)(9.81)\left(\frac{1}{50}+\frac{1}{100}\right)}$

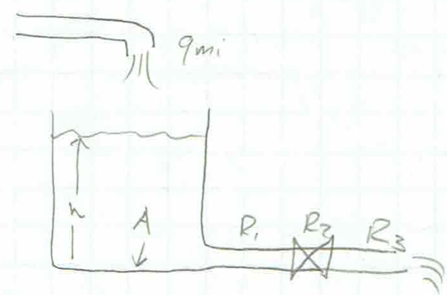
$= \dfrac{1896.3}{2000s^2 + 294.3s} = \dfrac{1896.3}{(2000)(.14715)}\left(\dfrac{.14715}{s^2 + s(.14715)}\right)$

$h(t) = 6.44\left(1 - e^{-.14715t}\right)$

$\boxed{\text{S.S. height} = 6.44\ m}$

2 For the water tank problem shown in 7.18, assume linear resistances and do the following...

a) $\boxed{\text{Inputs: } q_{mi} \quad \text{Outputs: } h \quad \text{parameters: } R_1, R_2, R_3, A, g, \rho}$



$R_{Tot} = R_1 + R_2 + R_3$

b) Write the diff eqs...

$$\boxed{\rho A \frac{dh}{dt} + \frac{\rho g}{R_{Tot}} h = q_{mi}}$$

c) Obtain the time constant, $\tau = \frac{\rho A R_{Tot}}{\rho g}$

$$\boxed{\tau = \frac{A R_{tot}}{g}}$$

d) Build the state space model. Indicate A, B, C, D if tank vol. + acc. Flow are needed.

$$\left[\frac{dh}{dt}\right] = \underbrace{\left[\frac{-g}{AR_{tot}}\right]}_{A}[h] + \underbrace{\left[\frac{1}{\rho A}\right]}_{B}[q_{mi}]$$

$V = Ah$

acc. Flow $= \rho A \frac{dh}{dt} = q_{mi} - \frac{\rho g}{R_{tot}} h$

$$\begin{bmatrix} V \\ A\frac{dh}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ \frac{-g}{R_{tot}} \end{bmatrix}}_{C}[h] + \underbrace{\begin{bmatrix} 0 \\ 1/\rho \end{bmatrix}}_{D}[q_{mi}]$$

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e) Find the corresponding TFs.

$$\boxed{\frac{H(s)}{Q_{mi}(s)} = \frac{1}{\rho A s + \rho g / R_{tot}}}$$

F) If $A = 3\,m^2$, $R_1 = R_3 = 20\,m^{-1}s^{-1}$, $R_2 = 100\,m^{-1}s^{-1}$, $q_{mi} = 5\,kg/s$, Find τ and S.S. value.

$Q(s) = \frac{5}{s}$

$R_{tot} = 140\,m^{-1}s^{-1}$

$$\tau = \frac{3(20+20+100)}{9.81} = \boxed{42.81\text{ sec} = \tau}$$

$$H(s) = \frac{5}{\rho A\left(\frac{g}{AR_{tot}}\right)} \frac{\frac{g}{A}R_{tot}}{s^2 + \frac{g}{AR_{tot}} s} = \frac{5}{\frac{\rho g}{R_{tot}}} \frac{\frac{g}{A}R_{tot}}{s\left(s + \frac{g}{A}R_{tot}\right)} = \frac{5}{70.07} \frac{.023}{s(s+.023)}$$

$h(t) = 0.0714(1 - e^{-.023t})$

$\boxed{\text{Steady State height: } 7.14\text{ cm}}$

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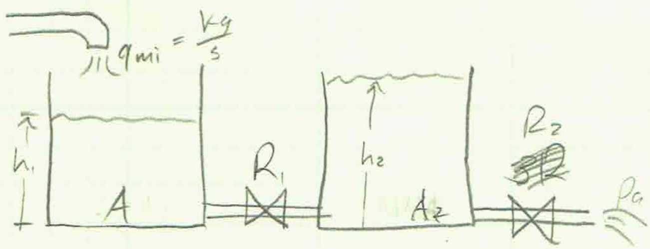
[3] # 7.23 → Dev. a model of two liquid heights in system shown in Fig P7.23.



$\rho A \frac{dh}{dt} = q$ $\qquad \frac{q_{mi}}{\rho} = \frac{\cancel{kg}}{s} \Big| \frac{m^3}{\cancel{kg}}$

(a)

$$\rho A \frac{dh_1}{dt} = q_{mi} - \frac{(h_1 - h_2)\rho g}{R} \longrightarrow \boxed{\rho A_1 \frac{dh_1}{dt} + \frac{\rho g}{R_1} h_1 = q_{mi} + \frac{\rho g}{R_1} h_2}$$

$$4\rho A \frac{dh_2}{dt} = \frac{(h_1 - h_2)\rho g}{R} - \frac{\rho g h_2}{3R} \longrightarrow \boxed{\rho A_2 \frac{dh_2}{dt} + \rho g h_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\rho g h_1}{R_1}}$$

(b) Using $R_1 = R$, $R_2 = 3R$, $A_1 = A$, $A_2 = 4A$, Find $\frac{H_2(s)}{Q_{mi}(s)}$

$$H_1(s)\left(\rho A s + \frac{\rho g}{R}\right) = Q_{mi}(s) + \frac{\rho g}{R} H_2(s)$$

$$H_2(s)\left(\rho A_2 s + \rho g\left(\frac{1}{R} + \frac{1}{3R}\right)\right) = \frac{\rho g}{R_1} H_1(s)$$

$$H_2(s) \frac{R_1}{\rho g}\left(\rho A_2 s + \rho g\left(\frac{1}{R} + \frac{1}{3R}\right)\right) = H_1(s)$$

$$H_2(s)\left[\left(\rho A s + \frac{\rho g}{R}\right)\left(\frac{R}{\rho g}\right)\left(\rho 4 A s + \rho g\left(\frac{1}{R} + \frac{1}{3R}\right)\right) - \frac{\rho g}{R}\right] = Q_{mi}(s)$$

$$\frac{H_2(s)}{Q_{mi}(s)} = \frac{1}{\left(\rho A s + \frac{\rho g}{R}\right)\left(\frac{4RA}{g}s + \frac{4}{3}\right) - \frac{\rho g}{R}} = \frac{1}{\frac{4\rho A^2 R s^2}{g} + \frac{4}{3}\rho A s + \frac{12\rho A}{3} s + \frac{4}{3}\frac{\rho g}{R} - \frac{\rho g}{R}\frac{3}{3}}$$

$$\boxed{\frac{H_2(s)}{Q_{mi}(s)} = \frac{1}{\frac{4\rho A^2 R}{g} s^2 + \frac{16\rho A}{3} s + \frac{\rho g}{3R}}}$$

$\frac{R_a}{g}$ $\qquad \frac{g\rho}{R_a}$

10

400

114.25 + 1

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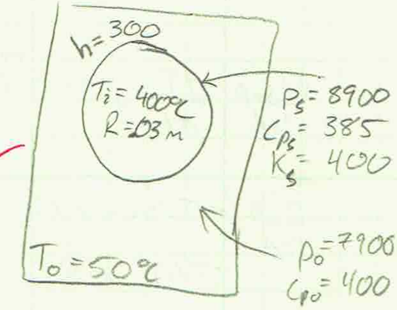
4 Using info from 7.57, do the following:



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a) Inputs: T_{si}, T_0
Outputs: T_s
Parameters: $h, r, \rho_0, C_{p0}, \rho_s, C_{ps}, K_s$

b) Write the corresponding diff eq.
conservation of energy, pg 442

$$C\frac{dT_s}{dt} = -\frac{1}{R}(T - T_b)$$

$$RC\frac{dT_s}{dt} + T_s = T_b$$

$$\boxed{114.2\frac{dT_s}{dt} + T_s = 50}$$

$C = mc_p = \rho V c_p$
$C = (8900)(1.131\times10^{-4})(385)$
$C = 387.5$
$R = \frac{1}{(300)(1.13\times10^{-2})} = 0.2947$

$RC = 114.2$

$$mc\frac{dT}{dt} = \frac{mc\,50}{\tau} - \frac{mcT_s}{\tau}$$

Assume T_0 doesn't change
$V_c = \frac{4}{3}\pi(.03)^3 = 1.131\times10^{-4}$
$A_c = 4\pi r^2 = 4\pi(.03)^2 = 1.131\times10^{-2}$

BLOCK →

c) Obtain the time constant. $\boxed{\tau = RC = 114.2\text{ s}}$

$mc_p\frac{dT}{dt}$

d) Build the SS model. Indicate A,B,C,D if T_s & power$_{sphere}$ is wanted.

$\dot{x} = Ax + Bu$
$y = Cx + Du$

$$\left[\frac{dT}{dt}\right] = \left[\frac{-1}{114.2}\right]\left[T_s\right] + \left[\frac{1}{114.2}\right]\left[50\right]$$

A, C, B, D

$$\begin{bmatrix} T_s \\ m_s c_{ps}\frac{dT}{dt} \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{m_s c_{ps}}{114.2} \end{bmatrix}\left[T_s\right] + \begin{bmatrix} 0 \\ \frac{m_s c_{ps}}{114.2} \end{bmatrix}\left[50\right]$$

Power$_{Stored}$ = 0 when $T_s = T_0$

$= \frac{mc(T_s - T_{si})}{t}$ (?)

where m=

$\frac{\rho V c_p}{Ah} = \frac{\cancel{mcp}}{Ah}$

$= Ah$

e) Find the temperature as a function of time

pg. 42

$$x(t) = x(0)\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] + \mathcal{L}^{-1}\left[\frac{F(s)}{s+a}\right]$$

$x(0) = 400°C$
$F(t) = 50/114.2$ $\quad F(s) = \frac{50}{114.2s}$
$a = \frac{1}{114.2}$

$$\dot{x} + ax = F(t) \rightarrow \frac{dT}{dt} + \frac{1}{114.2}T = \frac{50}{114.2}$$

$$= 400\mathcal{L}^{-1}\left(\frac{1}{s+1/114.2}\right) + \mathcal{L}^{-1}\left[\frac{50/114.2}{s^2+\frac{1}{114.2}s}\right] = 400e^{-at} + \mathcal{L}^{-1}\left[\frac{\frac{50}{114.2}}{\frac{1}{114.2}}\left(\frac{1/114.2}{s^2+\frac{1}{114.2}s}\right)\right]$$

$$= 400e^{-\frac{1}{114.2}t} + 50 - 50e^{-\frac{1}{114.2}t}$$

$$\boxed{T_s(t) = 50 + 350e^{-\frac{1}{114.2}t}}$$

$mc_p\frac{dT}{dt} = -\frac{1}{R}T_s +$

f) Estimate the time required to reach 120°C $\rightarrow \ln\left(\frac{70}{350}\right) = \frac{-1}{114.2}t$

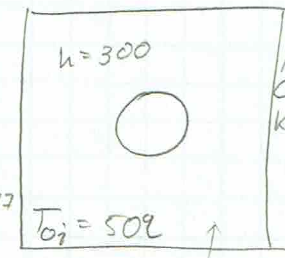
$t = 183.79$

$$\boxed{t = 183.8\text{ sec}}$$

$\boxed{5}$ Solve prob 7.58
Look @ prob 7.57. The oil volume is now $0.1 m^3$. oil is insulated to outside. Dev. a dynamic model of the system's T_s + T_o.

- $C_s = \rho V c_p = (8900)(\frac{4}{3}\pi(.03)^3)(385) = 387.5$
- $R_s = \frac{1}{(300)(1.13\times10^{-2})} = 0.295$
- $C_o = \rho V c_p = (7900)(0.1)(400) = 316{,}000$

$T_{s_i} = 400°C$
$R = 0.03 m$



$\rho_s = 8900$
$c_{ps} = 385$
$k_s = 400$

$\rho_o = 7900$
$c_{po} = 400$
$V_o = 0.1 m^3$

$$\boxed{\begin{aligned} R_sC_s\frac{dT_s}{dt} + T_s &= T_o \\ R_sC_o\frac{dT_o}{dt} + T_o &= T_s \end{aligned}}$$

$\tau_s = R_sC_s = 114.2$

$\tau_o = R_sC_o = 93125.2$

$R_s = \frac{1}{300\cdot1.13\times10^{-2}} = .2947$

$C_s = m_sC_{ps} = \rho_sV c_p = (8900)(1.13\times10^{-4})(385) = 387.5$

$C_o = (7900)(0.1)(400) = 316{,}000$

$T_s(s)R_sC_s s - T_{si}R_sC_s + T_s(s) = T_o(s)$

$T_o(s)R_sC_o s - T_{oi}R_sC_o + T_o(s) = T_s(s)$

$T_s(s)(R_sC_s s + 1) = T_o(s) + T_{si}R_sC_s$

$T_o(s) = \frac{T_s(s) + T_{oi}R_sC_o}{R_sC_o s + 1}$

$T_s(s)(R_sC_s s + 1) - \frac{T_s(s)}{sR_sC_o+1} = \frac{T_{oi}R_sC_o}{sR_sC_o+1} + T_{si}R_sC_s$

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$T_s(s)\left(R_sC_s s + 1 - \frac{1}{sR_sC_o+1}\right) = \frac{T_{oi}R_sC_o}{sR_sC_o+1} + T_{si}R_sC_s = \frac{T_{oi}}{s+\frac{1}{\tau_o}} + T_{si}\tau_s$

$$T_s(s) = \frac{T_{oi}/(s+\frac{1}{\tau_o})}{\tau_s s + 1 - \frac{1}{s\tau_o+1}} + \frac{T_{si}\tau_s}{\tau_s s + 1 - \frac{1}{s\tau_o+1}} =$$

$$= \frac{T_{oi}/(s+\frac{1}{\tau_o})}{\frac{\tau_s s(s\tau_o+1)}{(s\tau_o+1)} + \frac{(s\tau_o+1)}{(s\tau_o+1)} - \frac{1}{(s\tau_o+1)}} + \frac{T_{si}\tau_s}{\frac{\tau_s s(\tau_o s+1) + s\tau_o + 1 - 1}{s\tau_o+1}}$$

$$= \frac{T_{oi}}{\cancel{(s+\frac{1}{\tau_o})}}\,\frac{\cancel{(s+\frac{1}{\tau_o})}\tau_o}{(\tau_s\tau_o s^2 + s(\tau_s+\tau_o))} + \frac{T_{si}\tau_s(s\tau_o+1)}{\tau_s\tau_o s^2 + s(\tau_s+\tau_o)}$$

$$= \frac{T_{oi}\cancel{\tau_o}}{s\tau_s\cancel{\tau_o}(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})} + \frac{T_{si}\cancel{\tau_s}(s\tau_o+1)}{s\cancel{\tau_s}\tau_o(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})} = \frac{T_{oi}/\tau_s}{s(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})} + \frac{T_{si}\tau_o s + T_{si}}{s\tau_o(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})}$$

$$= \frac{T_{oi}/\tau_s}{s(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})} + \frac{T_{si}}{s+\frac{\tau_s+\tau_o}{\tau_s\tau_o}} + \frac{T_{si}/\tau_o}{s(s+\frac{\tau_s+\tau_o}{\tau_s\tau_o})} \rightarrow \frac{50/114.2}{.00876}\,\frac{.00876}{s(s+.00876)} + \frac{400}{1}\,\frac{1}{s+.00876} + \frac{.004295}{.00876}\,\frac{.00876}{s(s+.00876)}$$

$$\boxed{T_s(t) = 50.49 + 349.51e^{-.00876t}}$$

How long will it take to reach 130°C? 120°C?

The sphere will take 169 seconds to reach 130°C, and 184.4 seconds to reach 120°C.
This is less than 1 second longer to reach 120°C when holding the oil's temperature constant. Lumped capacitance is therefore useful.

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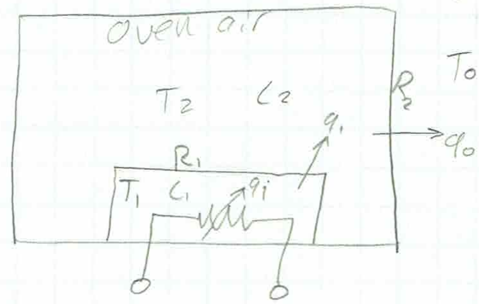
$\boxed{6}$ For the oven system shown in 7.54 do the following...

*Assume $T_o = \text{const.}$



a) Inputs: q_i, T_o
Outputs: T_2, T_1
Parameters: C_1, C_2, R_1, R_2

b) Write the corresponding diff eqs

$$q_1 = \frac{T_1 - T_2}{R_1}$$

$$\boxed{C_1 \frac{dT_1}{dt} = q_i - \left(\frac{T_1 - T_2}{R_1}\right)}$$

$$\boxed{C_2 \frac{dT_2}{dt} = \frac{T_1 - T_2}{R_1} - \frac{T_2 - T_o}{R_2}} = -T_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

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c) Build the state space model, indicate A, B

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{R_1C_2} & \frac{-1}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix}}_{\boxed{A}} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{R_2C_2} \end{bmatrix}}_{\boxed{B}} \begin{bmatrix} q_i \\ T_o \end{bmatrix}$$

d) Find the correspond TFs.

$$T_1(s)\left(C_1 s + \frac{1}{R_1}\right) = Q_i(s) + T_2(s)\frac{1}{R_1} \rightarrow T_2(s) = T_1(s)(C_1R_1 s + 1) - Q_i(s)R_1$$

$$T_2(s)\left(C_2 s + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right) = T_1(s)\frac{1}{R_1} + T_o(s)\frac{1}{R_2} \rightarrow \Big(T_1(s)\underbrace{(C_1R_1 s + 1)}_{Z_1} - \underbrace{Q_i(s)R_1}_{Z_2}\Big)\Big(\underbrace{C_2 s}_{Z_3} + \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}_{Z_4}\Big) - T_1(s)\frac{1}{R_1} = T_o(s)\frac{1}{R_2}$$

$$C_1C_2R_1 s^2 + C_2 s$$

$$T_1(s)Z_1Z_3 + T_1(s)Z_1Z_4 - Z_2Z_3 - Z_2Z_4 - T_1(s)\frac{1}{R_1} = T_o(s)\frac{1}{R_2}$$

$Q_i = 0$

$$T_1(s)\left(Z_1Z_3 + Z_1Z_4 - \frac{1}{R_1}\right) = \frac{T_o(s)}{R_2}$$

$T_o = 0$

$$T_1(s)\left(Z_1Z_3 + Z_1Z_4 - \frac{1}{R_1}\right) = Q_i\left(R_1C_2 s + R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right)$$

$$\frac{T_1(s)}{T_o(s)} = \frac{1/R_2}{C_1C_2R_1 s^2 + C_2 s + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)C_1R_1 s + \left(\cancel{\frac{1}{R_1}} + \frac{1}{R_2}\right) - \cancel{\frac{1}{R_1}}}$$

$$\frac{T_1(s)}{Q_i(s)} = \frac{R_1C_1 s + \left(1 + \frac{R_1}{R_2}\right)}{C_1C_2R_1 s^2 + s\left(C_2 + C_1 + C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}}$$

$$\boxed{\frac{T_1(s)}{T_o(s)} = \frac{1/R_2}{s^2 C_1C_2R_1 + s\left(C_1 + C_2 + C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}} \qquad \frac{T_1(s)}{Q_i(s)} = \frac{R_1C_1 s}{s^2 C_1C_2R_1 + s\left(C_1 + C_2 + C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}} + \frac{1 + \frac{R_1}{R_2}}{s^2 C_1C_2R_1 + s\left(C_2 + C_1 + C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}}}$$

cont.

$\boxed{6}$ cont.

d cont.)

$$\frac{T_2(s)}{T_0(s)} = \frac{s/R_2}{s^2 C_2 + s\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{s}{sC_1R_1^2+R_1}}$$

$$\frac{T_2(s)}{Q_i(s)} = \frac{1}{s^2 C_1C_2R_1 + s\left(C_1+C_2+C_1\frac{R_1}{R_2} - \frac{C_1}{sC_1R_1+1}\right) + \left(\frac{1}{R_1}+\frac{1}{R_2} - \frac{1}{sC_1R_1^2+R_1}\right)}$$

e) IF $q_i = 10\sin(.01t)$, Find $\mathcal{L}$

$$Q_i(s) = \frac{10 * .01}{s^2 + .01^2}$$

assume $T_0 = 20°C$

$$T_0(s) = \frac{20}{s}$$

$$T_1 = \frac{Q_i + T_2\frac{1}{R_1}}{C_1 s + \frac{1}{R_1}}$$

$$T_2\left(C_2 s + \left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right) = \frac{Q_i}{sC_1R_1+1} + \frac{T_2}{sC_1R_1^2+R_1} + \frac{T_0}{R_2}$$

$$T_2\left(C_2 s + \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{sC_1R_1^2+R_1}\right) = \frac{Q_i}{sC_1R_1+1} + \frac{T_0}{R_2}$$

$T_0 = 0$

$$\frac{T_2}{Q_i} = \frac{1}{s^2C_1C_2R_1 + C_2 s + sC_1 + \frac{1}{R_1} + \frac{sC_1R_1}{R_2} + \frac{1}{R_2} - \frac{sC_1R_1}{sC_1R_1+1} - \frac{1}{sC_1R_1^2+R_1}}$$

$Q_i = 0$

$$\frac{T_2}{T_0} = \frac{1/R_2}{sC_2 + \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{sC_1R_1^2+R_1}}$$

$$T_1(s) = \frac{20/R_2}{s^3C_1C_2R_1 + s^2\left(C_1+C_2+C_1\frac{R_1}{R_2}\right) + \frac{s}{R_2}} + \frac{(0.1)R_1C_1 s}{(s^2+.0001)\left(s^2C_1C_2R_1 + s\left(C_1+C_2+C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}\right)}$$

$$+ \frac{(0.1)(1+R_1/R_2)}{(s^2+.0001)\left(s^2C_1C_2R_1 + s\left(C_1+C_2+C_1\frac{R_1}{R_2}\right) + \frac{1}{R_2}\right)}$$

$$T_2(s) = \frac{20/R_2}{s^2C_2 + s\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{s}{sC_1R_1^2+R_1}} + \frac{(0.1)}{(s^2+.0001)\left(s^2C_1C_2R_1 + s\left(C_1+C_2+C_1\frac{R_1}{R_2} - \frac{C_1}{sC_1R_1+1}\right) + \left(\frac{1}{R_1}+\frac{1}{R_2} - \frac{1}{sC_1R_1^2+R_1}\right)\right)}$$