Fourier Analysis of a Voltage Signal Induced by Resonant Vibration

MCEN 3047 - Data and Measurements Laboratory

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Executive Summary

Resonant frequencies of a body describe how the body naturally vibrates or oscillates. Systems often experience vibrational impulses that may align with the resonant frequencies of that system. Depending on its purpose, it may be important to avoid or align outside frequencies with these natural frequencies.

The purpose of this laboratory is to identify the resonant frequencies of two aluminum bars using Fourier analysis. The bars are vibrated and a strain gauge records the vibrational behavior via voltage measurements in LabVIEW. Data is exported into MATLAB to undergo a Fourier analysis. A Fourier analysis of a signal decomposes the signal into smaller waves of various frequencies and amplitudes. Theoretical predictions are calculated based on the bar's properties and compared to the observed frequencies from the Fourier analysis. Uncertainties in the measurements and properties are used to calculate the total uncertainty in the theoretical frequencies.

The short bar's observed frequency deviated from the predicted value by 10.0%. The long bar's observed frequency deviated from the predicted value by 9.1%, 10.6%, or 40.9%, for the first three modes of vibration. All the observed frequencies fell within a 95% confidence interval of the theoretical values, except for the first mode of vibration in the long beam. The deviations in theoretical and observed values are sufficient to necessitate the use of a Fourier analysis.

Introduction

Resonance in a material body is a phenomenon that occurs when the body oscillates with the same frequency as the natural frequency of that body. The body's material and geometry determine its natural frequencies. When the frequency is matched by an outside force the amplitude of oscillation is maximized.

This laboratory determines the resonant frequencies of a thin aluminum bar held in a cantilever position at a certain length. These frequencies are found using strain gauges on the aluminum bar to output a frequency signal when the bar is put into oscillation. Then the data is analyzed with MATLAB to determine the dominant frequencies hidden amidst the noise. Following these methods, resonant frequencies of a body can be discovered by analyzing the natural vibrations the body produces.

Methods

To collect vibration data, strain gauges are mounted to the aluminum bars. The bars' specifications and corresponding uncertainties are listed in Table 1. When the bar is plucked, the changes in resistance are amplified with a Wheatstone bridge which outputs a change in voltage.

LabVIEW is used as the transducer to record the voltage oscillations with respect to time and then exports the raw data into MATLAB. The raw data collected is unfiltered and noisy, therefore Fourier analysis is used in MATLAB to minimize the noise and identify the dominant frequencies.

To verify the functionality of the Fourier analysis code, a test is ran using a constructed voltage signal. MATLAB is used to create a sine wave with a frequency of 50Hz, and amplitude of 1V for 1 second. The sampling frequency is set at 300Hz. Note that a minimum frequency, known as the Nyquist frequency, could be found at a sampling frequency of 100Hz. This is twice the value of the highest frequency present in the signal.

Table 1. Aluminum Bar Specifications.

6061-T6 Al	Specifications
$\boldsymbol{L_{short}}$	10.0625 ± 0.03125 in
$\boldsymbol{L_{long}}$	42.875 ± 0.03125 in
W	1.0 ± 0.014 in
t	0.0625 ± 0.006 in
$\boldsymbol{\rho}$	0.0975 ± 0.001365 lb/in^3 [1]
E	9195.34 ± 290.074 ksi [2]

A random number generator added noise to the signal by adding a vector of random values (~ N(0, 1)) to the signal. MATLAB's Fourier analysis function (DFT) is used on the two signals and then transformed into a power spectral density (PSD). The analysis could be performed manually using Eq. 1. Note the power spectral density is proportional to square root of the frequency amplitude.

$$X_k = \sum_{t=0}^{N-1} f(t) e^{-2\pi i k/N} \qquad \textbf{Eq. 1}$$

A theoretical resonant frequency can be calculated based on the geometry of an object. The first three theoretical resonant frequencies are calculated using Eq. 2. which correspond to the modes of vibration, $n = 1, 2, 3$.

$$f_n = \frac{{C_n}^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \qquad \textbf{Eq. 2}$$

Where f_n is the theoretical resonant frequency of mode $n = 1, 2, 3$, L is the length of the bar, E is the modulus of elasticity, I is the moment of inertia, ρ is the density of the material, A is the cross-sectional area of the bar, and C_n is the mode coefficient. The coefficients are $C_1 = 1.875$, $C_2 = 4.694$, and $C_3 = 7.855$ for frequency modes of $n = 1, 2, 3$ respectively.

The uncertainties for the variables in Eq. 2 are turned into standard deviations by assuming a 95% confidence interval and dividing out the corresponding z-value of 1.96. The standard deviations in the theoretical frequencies are calculated using Eq. 3, which follows the typical form for uncertainty propagation.

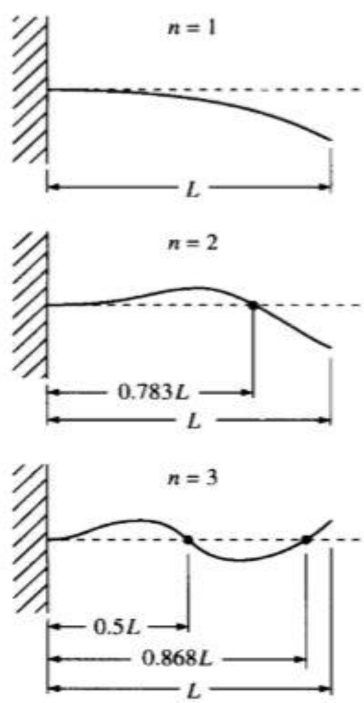

Figure 1. Nodes of resonant vibration in a cantilever beam [3].

Eq. 3

$$\sigma_{f_n} = \sqrt{\left(\frac{\partial f_n}{\partial L}\sigma_L\right)^2 + \left(\frac{\partial f_n}{\partial E}\sigma_E\right)^2 + \left(\frac{\partial f_n}{\partial \rho}\sigma_\rho\right)^2 + \left(\frac{\partial f_n}{\partial t}\sigma_t\right)^2}$$

Where $\frac{\partial f_n}{\partial [L,E,\rho,t]}$ are the partial derivatives of Eq. 2 with respect to each variable, and $\sigma_{[L,E,\rho,t]}$ are the standard deviations of the properties used in Eq. 2. The frequency standard deviation is then converted back to an uncertainty by multiplying it by the 95% confidence z-value, 1.96.

With each mode of vibration, there is a node present along the bar where the amplitude of oscillation is zero. These nodes may be predicted mathematically but they can also be observed visually on the beam while it is vibrating. Figure 1 shows the theoretical predictions of the node locations.

Results

The noisy test signal is shown in Fig. 2. The corresponding Fourier analysis for the noisy data is shown in Fig. 3. The Fourier analysis detected the test frequency applied to the signal, 50Hz.

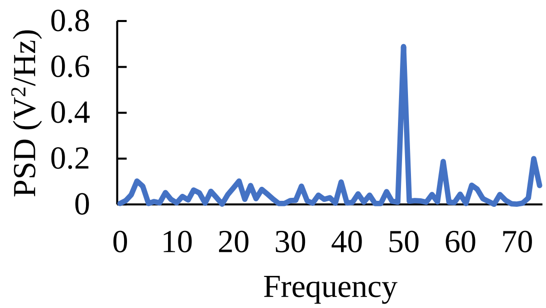

Figure 3. Power Spectral Density (V^2/Hz) as a function of frequencies for a noisy signal.

The voltage signal for the short bar is shown in Fig. 4. As shown in Fig. 5, the Fourier analysis on the short bar reveals a resonant frequency of 18Hz, corresponding to the first mode. As shown in Fig. 6, there are no distinguishable frequencies corresponding to the predicted second and third modes of vibration. Therefore, the short bar only exhibited the first mode of vibration.

The voltage signal for the long bar is shown in Fig. 7. Figure 8 shows a magnified view of the signal to show the low amplitude frequency. Figure 9 shows all three observed frequencies for the three observed modes in

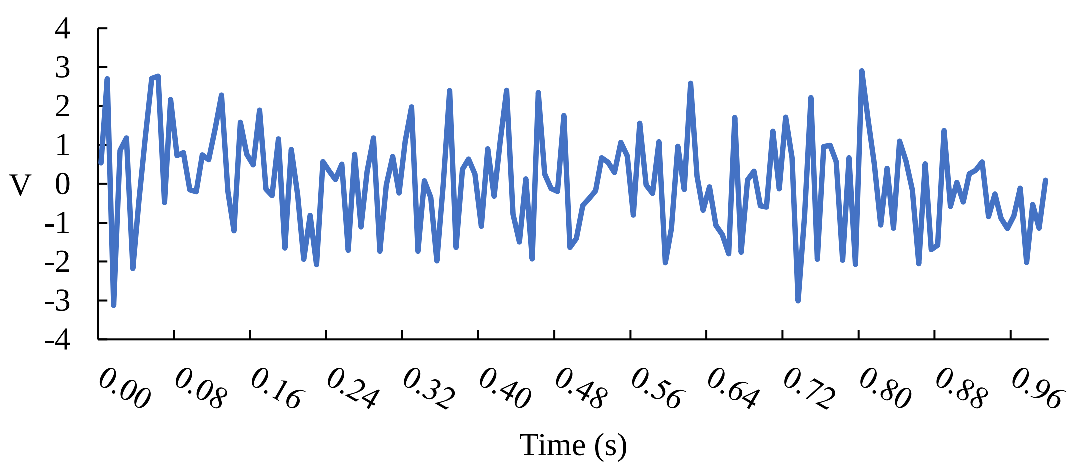

Figure 2. Noisy voltage (V) signal as a function of time (s).

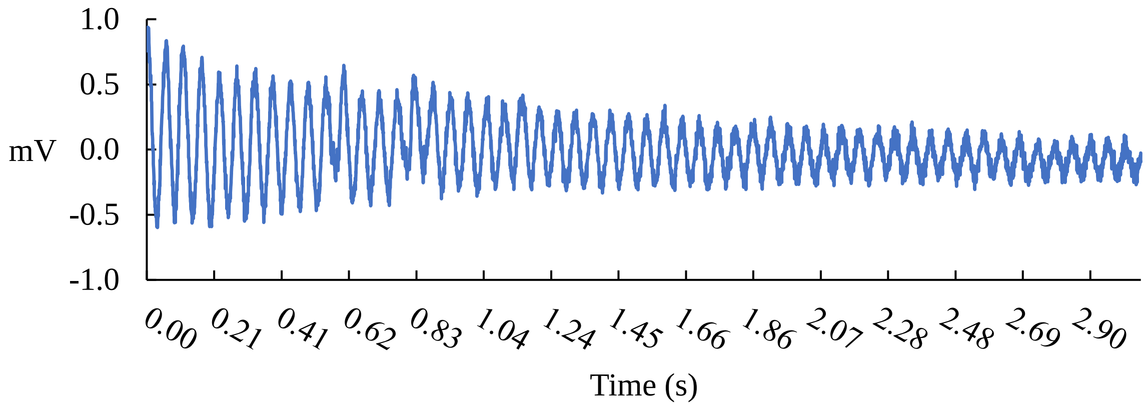

Figure 4. Voltage (mV) signal as a function of time (s) for a 10" aluminum bar.

table 2. Note that the amplitude of the third mode is too small to be visible while vibrating.

The observed frequencies are found by locating the max PSD in MATLAB for both lengths. In both cases, the frequencies are compared with theoretical calculations and a percent difference is computed. The results are shown in Table 2.

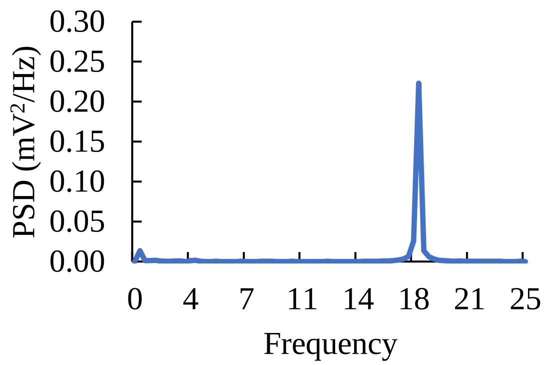

Figure 5. PSD (mV^2/Hz) as a function of frequency for the 10" bar.

Table 2. Frequency predictions and observations for short and long bar.

		Theoretical ($\boldsymbol{u_f}$ (95%))	Observed	% Diff
Short Bar	f_1	20.6 (4.0)	18.1	10.0
	f_2	126 (24.8)	-	-
	f_3	352 (69.4)	-	-
Long Bar	f_1	1.09 (0.21)	0.65	40.9
	f_2	6.86 (0.66)	6.13	10.6
	f_3	19.18 (3.62)	17.43	9.1

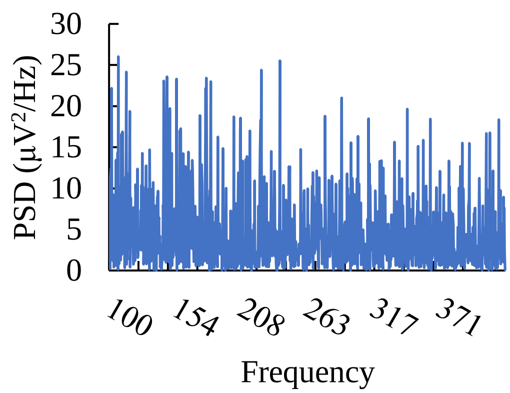

Figure 6. PSD (μV^2/Hz) as a function of frequency for the 10" bar.

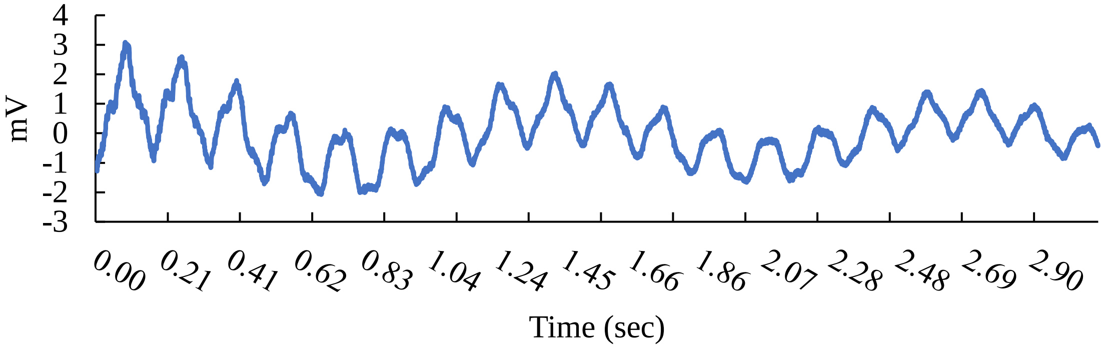

Figure 7. Voltage (mV) signal as a function of time (s) for a 42” aluminum bar.

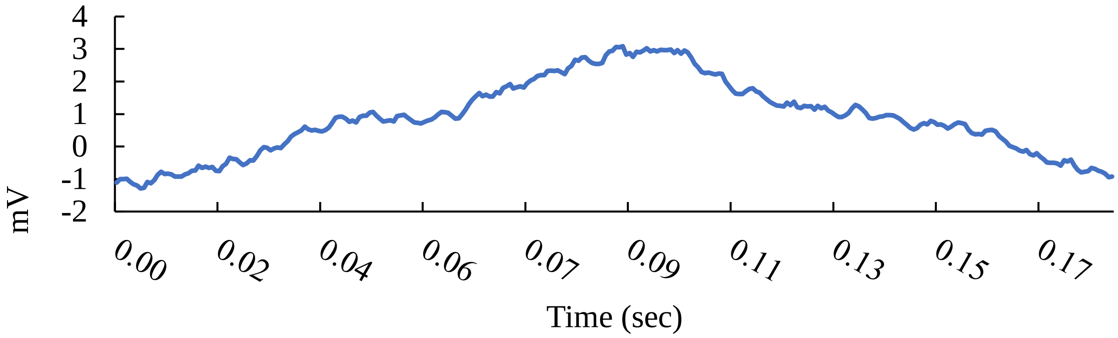

Figure 8. Voltage (mV) signal as a function of time (s) for a 42” aluminum bar. Magnified time to show low amplitude frequencies.

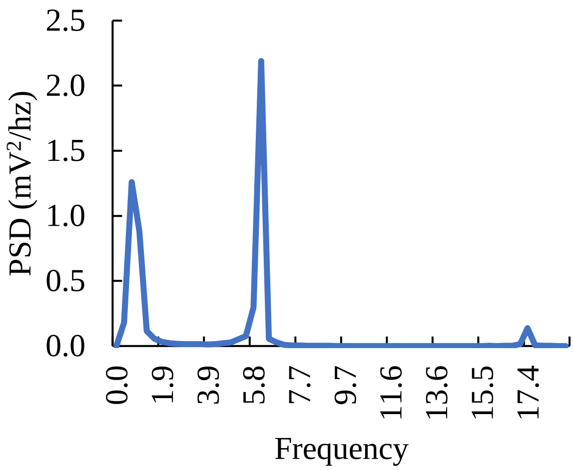

Figure 9. PSD (mV^2/Hz) as a function of frequency for the 42” aluminum bar.

References

[1] Davis, J. (2006). Experimental Measurement of the principle isentrope of aluminum 6061-T6 to 240 GPa [Abstract]. *Journal of Applied Physics,99*(103512), 2006th ser. http://dx.doi.org/10.1063/1.2196110

[2] Tsugawa, M. (2014). Lab 5 Elastic Modulus of Aluminum and Steel Beams. Retrieved April 11, 2017, from http://tinyurl.com/n8j7tqv

[3] Ronald F. Gibson. (1994). Principles of Composite Material Mechanics. New York, McGraw-Hill, Inc.