Exam 1

40/40

Name Ross Fischer

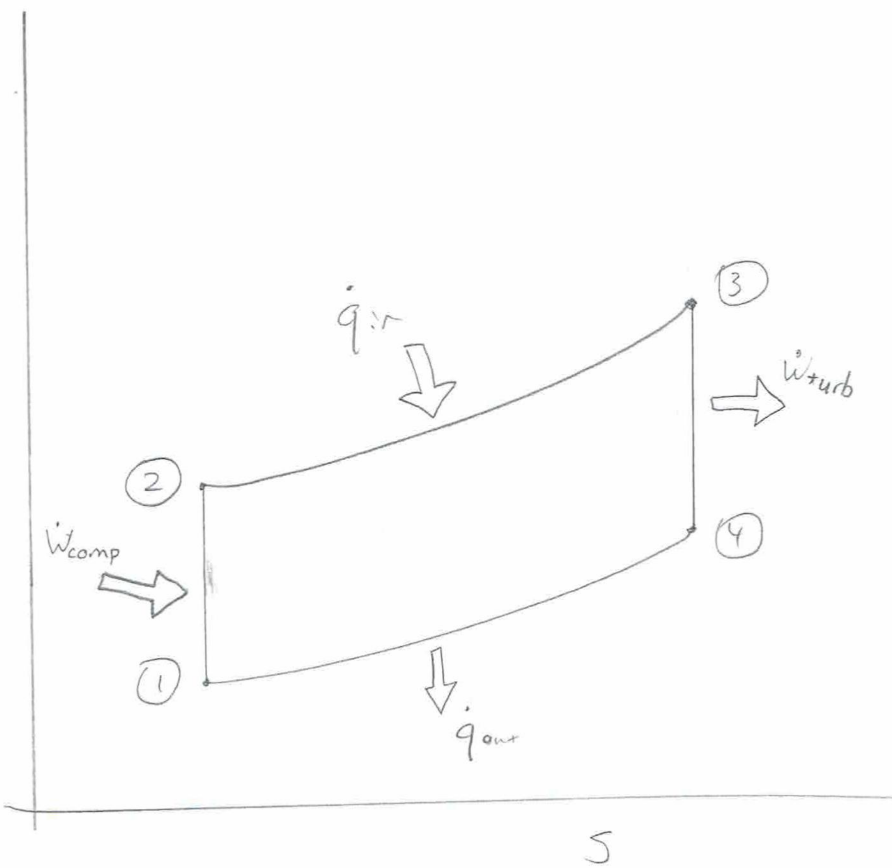
Exam is open book, but not open notes. You have 2 hours for this exam. Show all your work and mark your answers clearly!

40 points

$\frac{P_2}{P_1} = \frac{P_3}{P_4} = 19.1$

1. A gas turbine engine has a pressure ratio of 19.1 across the compressor and a temperature at the inlet to the turbine of 1110°C.
   (a) Draw a large T-s diagram of this cycle and label each state that you use in your calculations as well as work and heat interactions. **Use a straightedge to make your sketch. [6 points]**
   (b) Assuming constant specific heats and modeling this engine as an ideal Brayton cycle, determine the thermal efficiency of this engine. **[2 points]**
   (c) Using the cold-air standard assumptions and constant specific heats, determine the temperature (in °C) at each state in the cycle. **[6 points]**
   (d) Using the cold-air standard assumptions and constant specific heats, determine the mass flow rate of air (in kg/s) required to produce 100 MW of net power. **[2 points]**

(a)



1→2	Compressor
2→3	Heat Exchanger
3→4	Turbine
4→1	Heat Exchanger

$\dot{W}_{turb} = \dot{W}_{net} - \dot{W}_{comp}$

(b) Const. Spec. Heats $h \approx c_p T$ $\frac{T_2}{T_1} = \frac{P_2}{P_1}^{\frac{k-1}{k}}$

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}} \qquad r_p = \frac{P_2}{P_1} = 19.1 \quad K$$

$$= 1 - \frac{1}{19.1^{\frac{1.4-1}{1.4}}} = \boxed{56.9\,\%}$$

(c) cold air std assumptions + const. spec. heats det °C each state

1	2	3	4
$T_1 = 25°C$	$T_2 = 419°C$	$T_3 = 1110°C$	$T_4 = 322°C$

$$\frac{T_3}{T_4} = (r_p)^{\frac{k-1}{k}} \quad T_4 = T_3 / r_p^{\frac{k-1}{k}} = \frac{1383\,K}{19.1^{\frac{1.4-1}{1.4}}} = 595.4\,K = 322°C$$

$$\frac{T_2}{T_1} = (r_p)^{\frac{k-1}{k}} \quad T_2 = (19.1)^{\frac{1.4-1}{1.4}}(\underset{K}{298}) = 692.19\,K = 419.2°C$$

(d) det. $\dot{m}_{air}$ (kg/s) req. For 100 MW net power $C_{p\,air} = 1.005 \frac{kJ}{kg\,K}$

$\Delta T\,°C = \Delta T\,K$

$$\dot{W}_{net} = \dot{m}_{air}\left[(h_3 - h_4) - (h_2 - h_1)\right] \approx \dot{m}_{air} C_p\left[(T_3 - T_4) - (T_2 - T_1)\right]$$

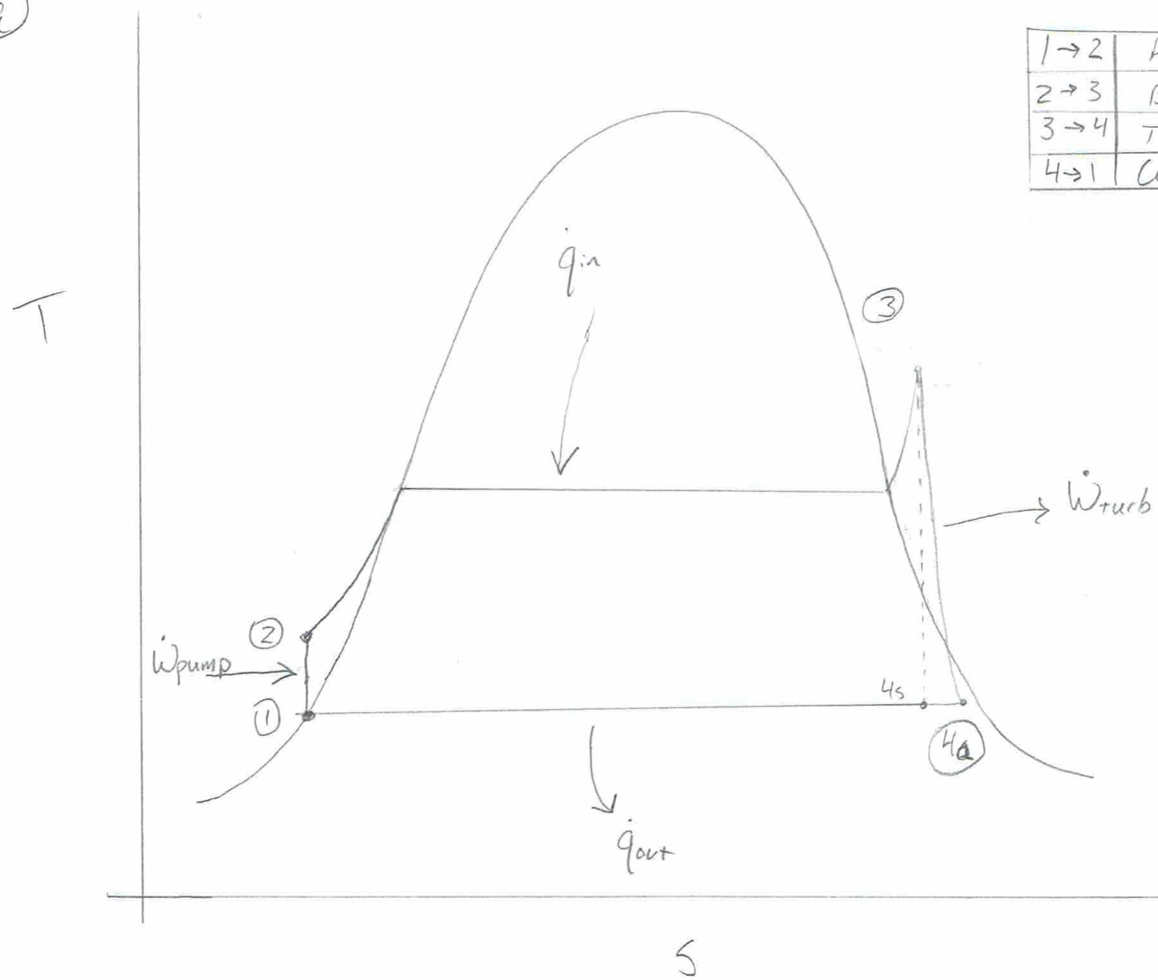
$$\left[(1110 - 322) - (419 - 25)\right] = 394$$

$$\dot{m}_{air} = \frac{100 \times 10^3}{1.005\,(394)} \frac{\cancel{kW}}{1} \frac{kg\,K}{\cancel{kJ}} \frac{1}{K} \frac{\cancel{kJ}}{s}$$

$$\boxed{\dot{m}_{air} = 252.5 \frac{kg}{s}}$$

2. A power plant has a boiler that produces steam at 1250 psia and 850°F. After passing through a turbine with an isentropic efficiency of 85%, the steam is condensed at a pressure of 15 psia back to a saturated liquid state. Model this power plant using a Rankine cycle.
   (a) Draw a large T-s diagram of this cycle and label each state that you use in your calculations as well as work and heat interactions. **Use a straightedge to make your sketch**. **[6 points]**
   (b) Determine the enthalpy at each state in the cycle. **[12 points]**
   (c) What is the quality of steam leaving the turbine? **[2 points]**
   (d) What is the efficiency of this cycle? **[2 points]**
   (e) What mass flow rate of steam (in lbm/h) is required to produce 25 MW of net power? **[2 points]**

a



1 → 2	Pump
2 → 3	Boiler
3 → 4	Turbine
4 → 1	Compresser

$W_{turb} = \dot{W}_{net} - \dot{W}_{comp}$

(b)

1 sat. liq.

$\boxed{h_1 = 181.2 \text{ Btu/lbm}}$ ✓

$s_1 = .3137$
$P_1 = 15$ psia

2 comp. liq.

$\boxed{h_2 = 185.4 \text{ Btu/lbm}}$ ✓

$s_1 = s_2 = .3137$
$P_2 = 1250$ psia

3

$\boxed{h_3 = 1407.7 \text{ Btu/lbm}}$ ✓

$T_3 = 850°F = 1310 R$
$P_3 = 1250$ psia

4 actual

$\boxed{h_{4a} = 1075.4 \text{ Btu/lbm}}$ ✓

$P_4 = 15$ psia

Boiler → $P_2 = P_3 = 1250$ psia

$T_3 = 850°F = 1310R$

Condenser → $P_4 = P_1 = 15$ psia

2

$S_2 = .3137$

1000 psia: $.283 = \frac{h - 170.4}{220.7 - 170.4}$, $h_{2,1} = 184.6$

1500 psia: $.292 = \frac{h - 171.5}{221.7 - 171.5}$, $h_{2,2} = 186.17$

$h_2 = 185.39$

3

$\frac{50}{100} = \frac{h - 1376.4}{1439 - 1376.4}$

$h_3 = 1407.7$

$.5 = \frac{s - 1.5347}{1.5826 - 1.5347}$

$s_3 = 1.55865$

4s

$s_{4s} = s_3 = 1.55865$
$P_4 = 15$ psia

$\frac{1.55865 - .31370}{1.44441} = \frac{h_{4s} - 181.21}{969.47}$

$x_s = .8619$ $\quad h_{4s} = 1016.8 \frac{\text{Btu}}{\text{lbm}}$

4a

$\eta_T = .85 = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$

$.85 = \frac{(1407.7 - h_{4a})}{(1407.7 - 1016.8)}$

$332.265 = 1407.7 - h_{4a}$

$h_{4a} = 1075.435$

(c) quality?

$$x = \frac{h_{4a} - h_{f@15psia}}{h_{fg@15psia}} = \frac{1075.4 - 181.21}{969.47} = \boxed{92\% = x}$$

(d) $$\eta = \frac{\text{desired out}}{\text{req in}} = \frac{(h_3 - h_{4a}) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(1407.7 - 1075.4) - (185.4 - 181.21)}{(1407.7 - 185.4)} = \boxed{26.84\%}$$

(e) $\dot{m}_{steam}$ (lbm/h) req. For 25 MW_{net} Power?

$$25 \times 10^3 \cancel{kW} \Big| \frac{3412.14 \ \frac{Btu}{h}}{1 \ \cancel{kW}}$$

$$\dot{W}_{net} = \dot{m}_{steam}\left[(h_3 - h_{4a}) - (h_2 - h_1)\right] = \dot{m}\left[(1407.7 - 1075.4) - (185.4 - 181.2)\right] \qquad 25 \times 10^3 kW = 85{,}303{,}500 \ \frac{Btu}{h}$$

$$\dot{m}_{steam} = \frac{85{,}303{,}500}{328} \ \frac{\cancel{Btu}}{h} \ \frac{lbm}{\cancel{Btu}} = \boxed{259{,}992 \ \frac{lbm}{h}}$$