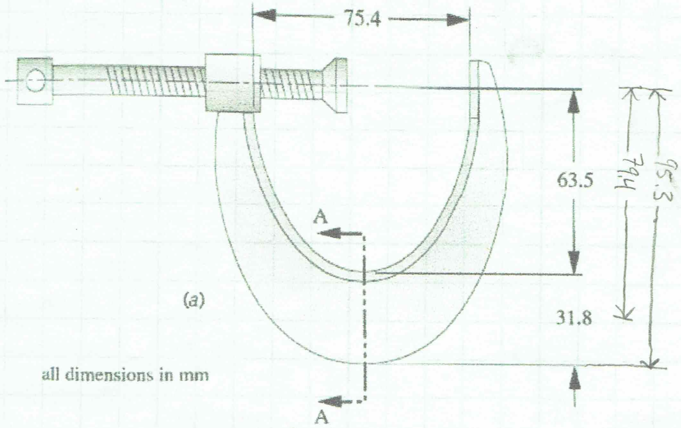
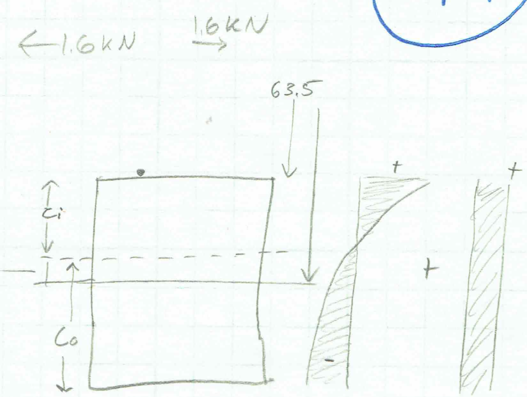
99

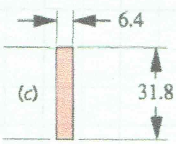
5-70 A C-clamp as shown in Figure P5-24a has a rectangular cross section as in Figure P5-24c. Find the static factor of safety if the clamping force is 1.6 kN and the material is class 50 gray cast iron.





$I = \frac{bh^3}{12} = \frac{(.0064)(.0318)^3}{12}$

$I = 1.715 \times 10^{-8}$



$M = Fr = (1600)(.0794) = 127.04\ Nm$

$A = bh = 203.52\ mm^2$

$A = 2.035 \times 10^{-4}$

$S_{uT} = 359\ MPa$

$S_c = 1131\ MPa$

$A = b\Delta r \qquad \frac{dA}{dr} = b$

$$\frac{A}{\int \frac{dA}{r}} \quad \int_{63.5}^{95.3} \frac{b}{r}\,dr = b\ln(r)\Big|_{63.5\,mm}^{r=95.3\,mm} \Rightarrow \frac{A}{\int\frac{dA}{r}} = \frac{2.035\times10^{-4}}{(.0064)(\ln(.0953)-\ln(.0635))} = 0.0783\,m$$

15

$e = 79.4\,mm - 78.3\,mm = 1.073\ mm$

$c_i = \frac{31.8}{2} - 1.073 = 14.827\ mm$

$c_o = 15.9 + 1.073 = 16.97\ mm$

$\underbrace{\sigma_1 = \quad \sigma_2 = 0,\ \sigma_3}$

$$\sigma_i = \frac{M}{eA}\left(\frac{c_i}{r_i}\right) + \frac{F}{A} = \frac{127.04}{(1.07\times10^{-3})(2.035\times10^{-4})}\left(\frac{.014827}{.0635}\right) + \frac{1600}{2.035\times10^{-4}} = 144\ MPa$$

$$\sigma_o = \frac{-M}{eA}\left(\frac{c_o}{r_o}\right) + \frac{F}{A} = \frac{-127.04}{(1.07\times10^{-3})(2.035\times10^{-4})}\left(\frac{16.97}{95.3}\right) + \frac{1600}{2.035\times10^{-4}} = -96\ MPa$$

$\sigma_1 =$

$N = \frac{S_{uT}}{\tilde{\sigma}} = \frac{359}{144} = 2.49$ → Use "C"s

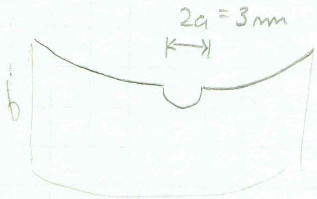
$\boxed{N = 2.49}$

5-76 Assume that the curved beam of Problem 5-70 has a crack on its inside surface of half-width $a = 1.5$ mm and a fracture toughness of 35 MPa-m $^{0.5}$. What is its safety factor against sudden fracture?

?

1.5

$\sigma_{nom} = \tilde{\sigma} = 144$ MPa



$b = t/2$

(-1)

$\frac{14}{15}$

$\frac{a}{b} = \frac{1.5}{31.6/2} = .047 \rightarrow \frac{a}{b} < 0.4$, so use eq 5.14d, $\beta = 1.12$

$K = \beta \sigma_{nom} \sqrt{\pi a} = (1.12)(144\ \text{MPa})\sqrt{\pi(.0015)} = 11.07\ \text{MPa}\sqrt{m}$

$N_{FM} = \frac{K_c}{K} = \frac{35}{11.07} = 3.16$ $\boxed{N_{FM} = 3.16}$

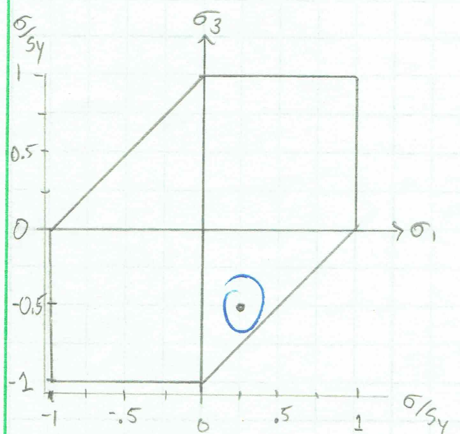
5-80 A differential element is subjected to the stresses (in MPa): $\sigma_1 = 70$, $\sigma_2 = 0$, $\sigma_3 = -140$. A ductile material has the strengths (in MPA): $S_{ut} = 350$, $S_y = 280$, $S_{uc} = 350$. Calculate the safety factor and draw σ_1 - σ_3 diagrams of each theory showing the stress state using:

(a) Maximum shear stress theory. → hexagon
(b) Distortion energy theory. → ellipse

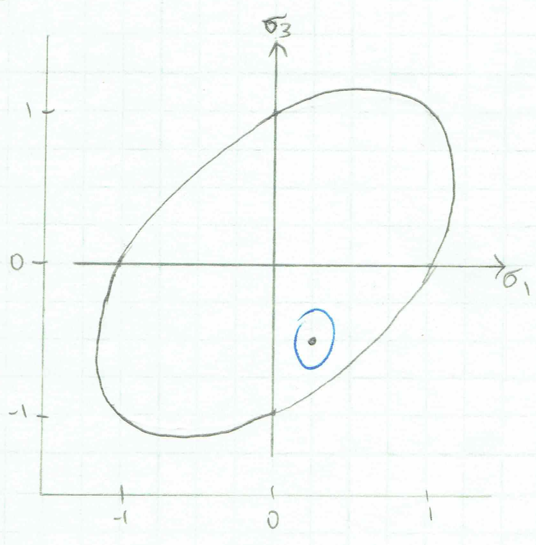
$\sigma_1 = 70$ $\quad S_{UT} = 350$
$\sigma_2 = 0$ $\quad S_{UC} = 350$
$\sigma_3 = -140$ $\quad S_Y = 280$

(a) $N = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{280}{70+140} = 1.33$ $\quad \boxed{N = 1.33}$

15



(b) $S_Y/N = \dfrac{280}{N} = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2} = \sqrt{70^2 - 70{\cdot}140 + (-140)^2}$ $\quad \boxed{N = 1.51}$



5-81 A part has the combined stress state and strengths given (in MPa) of: $\sigma_x = 70$, $\sigma_y = 35$, $\tau_{xy} = 31.5$, $S_{ut} = 140$, $S_{uc} = 140$, $S_y = 126$. Using the Distortion-Energy failure theory, find the von Mises effective stress and factor of safety against static failure.

$\sigma_x = 70$ $\quad S_{uT} = 140$
$\sigma_y = 35$ $\quad S_{uc} = 140$
$\tau_{xy} = 31.5$ $\quad S_y = 126$

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = \sqrt{70^2 + 35^2 - 70(35) + 3(31.5)^2}$$

$\boxed{\sigma' = 81.6 \text{ MPa} \quad N = 1.54}$

$N = S_y/\sigma' = 126/81.6$

15

5-85 A part has the combined stress state and strengths given (in MPa) of: $\sigma_x = 70$, $\sigma_y = 35$, $\tau_{xy} = 31.5$, $S_{ut} = 140$, $S_{uc} = 560$, $S_y = 126$. Using the Modified-Mohr failure theory, find the effective stress and factor of safety against static failure.

$C = \frac{\sigma_x + \sigma_y}{2} = \frac{70+35}{2} = 52.5$

$R = \sqrt{\left(\frac{70-35}{2}\right)^2 + 31.5^2} = 36$

$\sigma_1 = 88.53$

$\sigma_2 = 0$

$\sigma_3 = 16.47$

$C_1 = \frac{1}{2}\left[88.53 + \frac{2 \cdot 140 - 560}{-560}(88.53)\right] = 66.39$

$C_2 = \frac{1}{2}\left[-16.47 + \frac{2 \cdot 140 - 560}{-560}(16.47)\right] = -4.11$

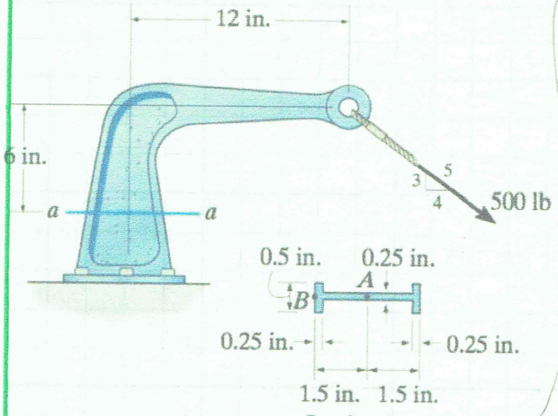
$C_3 = 1/2\left[-72.07 + \frac{2 \cdot 140 - 560}{560}(105)\right] = -9.79$

$\boxed{\tilde{\sigma} = 88.53 \text{ MPa} \quad N = 1.58}$

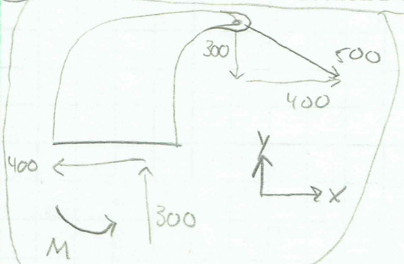
15

$N = S_{UT}/\tilde{\sigma} = 140/88.53 = 1.58$

6. (25 Points) For points A and B, do the following:
   a. If the material is aluminum, sketch the failure envelope using both theories. Select the material, clearly identify the relevant points and calculate the safety factors.
   b. If the material is gray cast iron, sketch the failure envelope using the modified-Mohr theory. Select the material, clearly identify the relevant points and calculate the safety factors.



Section *a–a*



$\sum M_{a\text{-}a} = 0 = +M - 300(12) - 400(6)$

$M = 6000$ in·lbs

$A = 2(.25\cdot.5) + .25\cdot2.5 = 0.875\ in^2$

$I_{1,2,3} = 2\left(\frac{1}{12}(.5)(.25)^3\right) + \frac{1}{12}(.25)(2.5)^3)$

$I_{total} = \sum(I_i + A_i d_i^2) = 0.799$

$\sigma_x = 0$

$\sigma_y = \frac{-300}{.875} = -342.9$ psi X

$\sigma_{bend} = \frac{Mc}{I} = \frac{6000(1.5)}{.799} = 11,257$ psi ✓

$\tau_{bend} = V/A_{web} = \frac{400}{.625} = 640$ psi ~

$\sigma_x = \sigma_{bend} - \sigma_{F,axial}$

Pt A

$\sigma_{y_{tot}} = \sigma_y = 342.9$

$\tau = \tau_{bend} = .640$

$C = \frac{\sigma_y}{2} = 171.45$

$R = \sqrt{\frac{\sigma_y}{2} + \tau^2} = 662.6$ psi

$\sigma_1 = 834$
$\sigma_2 = 0$
$\sigma_3 = -491$

Pt B

$\sigma_{y_{tot}} = \sigma_y + \sigma_{bend} = 10914$

$\tau = 0$

$C = 5457$
$R = 5457$

$\sigma_1 = 10914$
$\sigma_2 = 0$
$\sigma_3 = 0$

A - Aluminum (1100 sheet anneled)

$S_{UT} = 13000$ psi $S_y = 5000$ psi

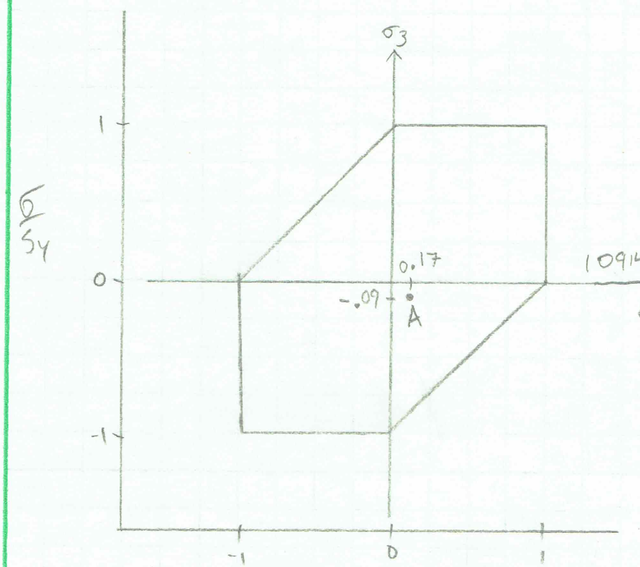
Gray Cast iron   class 20

$U_{TS} = 22$ kpsi

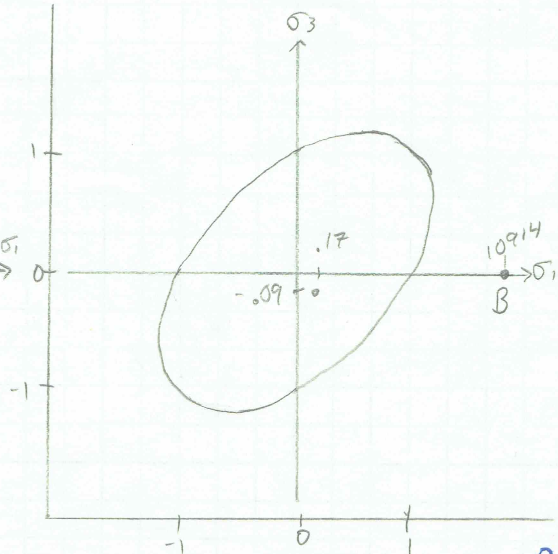
$S_{uc} = 83$ kpsi

25

Max shear Stress hexagon



Distortion Energy Ellipse



$N_A = \dfrac{S_Y}{\sigma_1 - \sigma_3} = \dfrac{5000}{834+491} = \boxed{3.77 = N_A}$

$N_B = \dfrac{5000}{10914} = 0.458$ $\boxed{0.458 = N_B}$

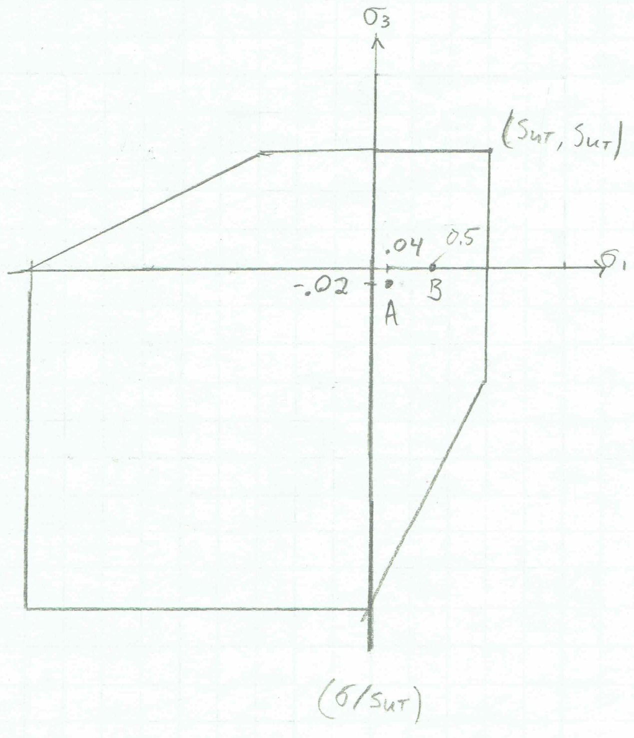
$\boxed{N_A = 4.31 \quad N_B = 0.458}$

Procedure ok

$N = S_Y / \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$

$N_A = 5000/\sqrt{834^2 + 834 \cdot 491 + 491^2}$

$N_B = 5000/\sqrt{10914^2}$



$N_A = \dfrac{S_{UT}}{\sigma_1} = \dfrac{22000}{834} = 26.4$

$N_B = \dfrac{22000}{10914} = 2.01$

$\boxed{N_A = 26.4 \quad N_B = 2.01}$