SPRING 2017 – EXAM 2
Wednesday, March 8, 2017

Name: Fischer

For all problems, state the assumptions and write complete sentences for explanations.

PROBLEM 4 (35 Points)

The link shown in the Figure below represents a model of the suspension system for the a certain FSAE car. All dimensions are shown in inches. All elements have a hollow circular cross section where the outside diameter is ½ inches and the inside diameter is ¼ inches. The joints are made of 1/8-inch pins working in double shear The ground reaction force W is 500 lb. Assume steel for all elements.

Do the following:

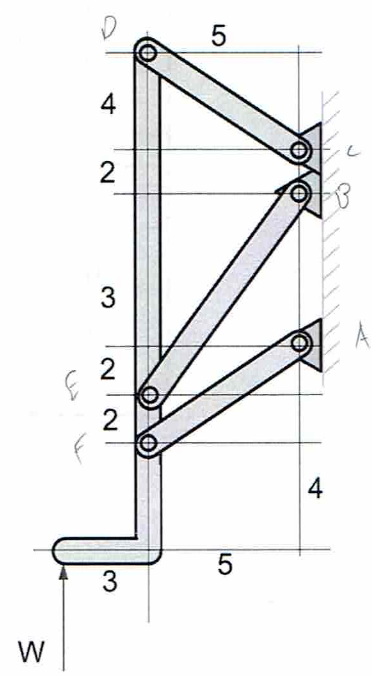
a. Calculate all the reaction forces and all the forces on the members.
b. Verify the answers by providing an FBD showing all the forces involved.
c. Obtain the maximum shear stress at the pins.
d. Identify the non-vertical elements under compression loads and obtain their critical load.
e. State whether the non-vertical element will buckle or not. Explain.

For the vertical L-shaped member do the following:

f. Obtain the shear force and bending moment using singularity functions.
g. Draw the shear force and bending moment diagrams.
h. Identify the region with the largest shear force and obtain the maximum shear stress.
i. Identify the region with the largest bending moment and obtain the normal stress due to bending.
j. Include the effect of the 1/8-inch holes to correct the value obtained in the previous part. Use graphs from the textbook.
k. If possible, select a steel material that will withstand the previous calculated load.

If a 10-inch diameter tire is attached to the end of the L-shaped member and the coefficient of kinetic friction is 0.25, do the following:

l. Calculate the torque acting on the horizontal part of the member.
m. Calculate the shear stress on the member due to the torque.
n. Obtain the angular deformation of the member.



PROBLEM 5 (10 Points)

Obtain the safety factor for the skin of a 4-engine Airbus airplane by doing the following:

a. Obtain the corresponding dimensions.
b. Identify the model theory needed to calculate stresses. Explain why this was selected.
c. Estimate the associated pressure.
d. Calculate the relevant stress(es). Explain why they were chosen.
e. Compare the stress(es) to the yield strength of the material.

PROBLEM 5 (15 Points)

Pipe bursting at low temperatures is a common problem due to the pressure that ice exert on pipes. The chart below shows the dimensions for Schedule 80 PVC dimensions. Perform the analysis for a pipe with a nominal size of 3/8 inches.

Do the following:

a. Obtain the corresponding dimensions.
b. Identify the model theory needed to calculate stresses. Explain why this was selected.
c. Obtain the maximum radial stress that can be applied to the pipe under yielding.
d. Obtain the maximum tangential stress that can be applied to the pipe under yielding.
e. Obtain the maximum axial stress that can be applied to the pipe under yielding.
f. Compare the values obtained in parts d, e ~~and f~~ to the recommended maximum water pressure.

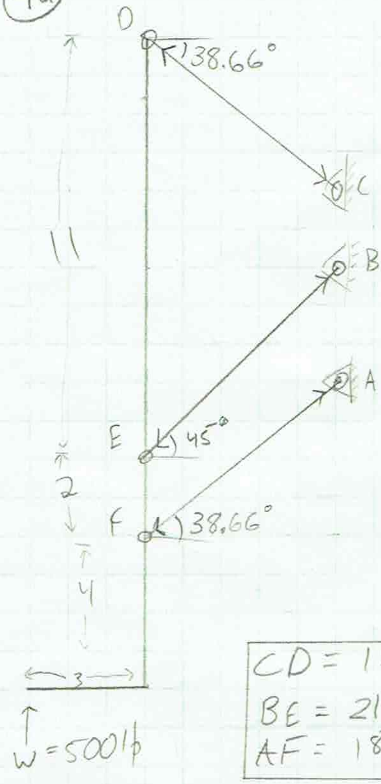
c, d, + e

Schedule 80 PVC Pipe Dimensions

Nominal Pipe Size (in)	O.D.	Average I.D.	Min. Wall	Nominal Wt./ft.	Maximum W.P. PSI*
1/8	0.405	0.195	0.095	0.068	1230
1/4	0.540	0.282	0.119	0.115	1130
3/8	0.675	0.403	0.126	0.158	920
1/2	0.840	0.526	0.147	0.232	850
3/4	1.050	0.722	0.154	0.314	690
1	1.315	0.936	0.179	0.461	630
1-1/4	1.660	1.255	0.191	0.638	520
1-1/2	1.900	1.476	0.200	0.773	470
2	2.375	1.913	0.218	1.070	400
2-1/2	2.875	2.29	0.276	1.632	420
3	3.500	2.864	0.300	2.186	370
4	4.500	3.786	0.337	3.196	320
6	6.625	5.709	0.432	6.102	280
8	8.625	7.565	0.500	9.269	250
10	10.750	9.493	0.593	13.744	230
12	12.750	11.294	0.687	18.909	230
14	14.000	12.41	0.750	22.681	220
16	16.000	14.213	0.843	29.162	220
18	18.000	16.014	0.937	36.487	220
20	20.000	17.814	1.031	44.648	220
24	24.000	21.418	1.218	63.341	210

Ross Fischer
EXAM 2 - TAKE HOME

4a



$\Sigma F_y = 0 = W + CD\sin(38.7) - BE\sin(45) - AF\sin(38.7)$

$\Sigma F_x = 0 = \quad - CD\cos(38.7) - BE\cos(45) - AF\cos(38.7)$

$\Sigma \overset{\curvearrowleft}{M}_D = 0 = -500 \cdot 3 \quad -(11)BE\cos(45) - AF\cos(38.7)(13)$

$$\begin{array}{cccc} CD & BE & AF & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} \sin(38.7) & -\sin(45) & -\sin(38.7) & -500 \\ -\cos(38.7) & -\cos(45) & -\cos(38.7) & 0 \\ 0 & -(11)\cos(45) & -(13)\cos(38.7) & 1500 \end{array}\right]$$

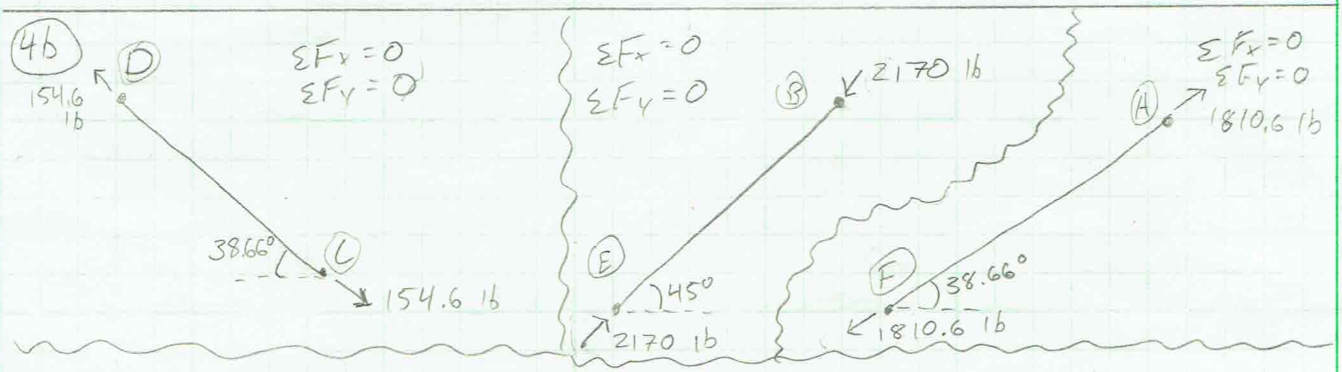
RREF on calc

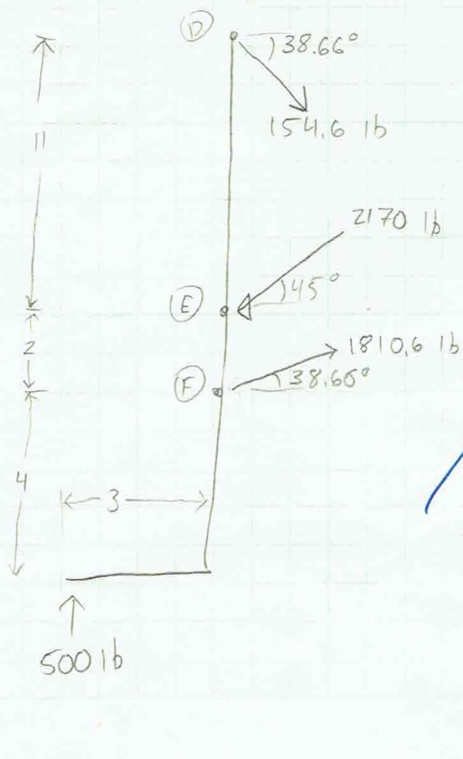
$CD = -154.56$; $BE = 2170.11$; $AF = -1810.56$

$$\boxed{\begin{array}{ll} CD = 154.6 \text{ lb} & \text{Tension} \\ BE = 2170.1 \text{ lb} & \text{Compression} \\ AF = 1810.6 \text{ lb} & \text{Tension} \end{array}}$$

31/35

4b





$\Sigma F_x = 0 = 154.6\cos(38.7) - 2170\cos(45) + 1810 \cdot (\cos 38.7)$

$0 = 0$ ✓

$\Sigma F_y = 0 = -154.6\sin(38.66) - 2170\sin(45) + 1810(\sin 38.66) + 500$

$0 = 0$ ✓

(4c) Pin D

$\tau = \frac{4}{3}\frac{F}{2A}$

$A = \pi d^2/4 = \pi\left(\frac{1''}{8}\right)^2/4$

$A = .0123 \text{ in}^2$

Missing Factor

-1

$\tau_D = \frac{P_D/2}{A} = \frac{154.6/2 \text{ lb}}{.0123''} = 6299$

$\boxed{\tau_D = 6299 \text{ psi}}$

Pin E

$\tau_E = \frac{2170.1/2 \text{ lb}}{.0123 \text{ in}^2} \Rightarrow \boxed{\tau_E = 88{,}418 \text{ psi}} \times \frac{4}{3}$

Pin F

$\tau_F = \frac{1810.6/2 \text{ lb}}{.0123 \text{ in}^2} \boxed{\tau_F = 73{,}771} \times \frac{4}{3}$

Not the same

(4d)

Compression $\longrightarrow$ BE = 2170.1 lb

link BE

$P_{CR} = \frac{\pi^2 E I}{l^2} = \frac{\pi^2(29\times10^6)(.002876)}{\sqrt{50}\cdot\sqrt{50}} \frac{\text{lb}}{\cancel{\text{in}^2}} \frac{\cancel{\text{in}^4}}{\cancel{\text{in}^2}}$

$\boxed{P_{CR} = 16{,}463 \text{ lb}}$ X

Area: $\frac{\pi}{4}\left(d_o^2 - d_i^2\right)$

BE

$L = L_{eff} = \sqrt{50}$

$I = \frac{\pi}{64}\left(.5^4 - .25^4\right) = 0.002876 \text{ in}^4$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{.002876}{.0123}} = .4836$ X

$S_r = \frac{l}{k} = \frac{\sqrt{50}}{.4836} = 14.622$

$\downarrow$

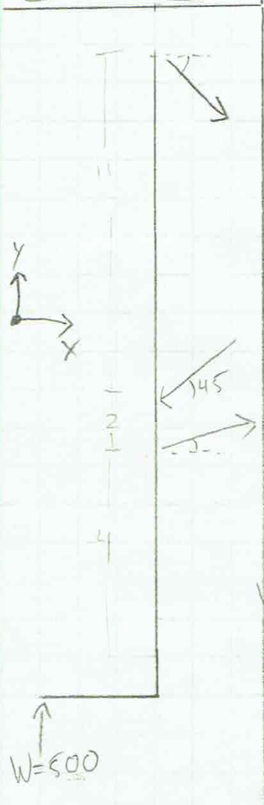
long

-2

$E_{steel} = 29\times10^6 \text{ Psi}$

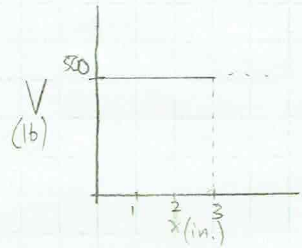
(4e) The non vertical element will not buckle because the compressive load, 2170 lbs, is less than the critical load, 16,463 lbs.

4F + 4g

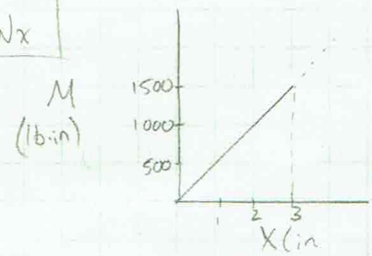


x-direction $0 < x < 3$

$\boxed{V = W}$



$\boxed{M = Wx}$



$q(x) = ??$

y-direction $0 < y < 17$

shear

$V = -1810.6\cos(38.66)\langle x-4\rangle^0 + 2170.1\cos(38.66)\langle x-6\rangle^0 - 154\cos$

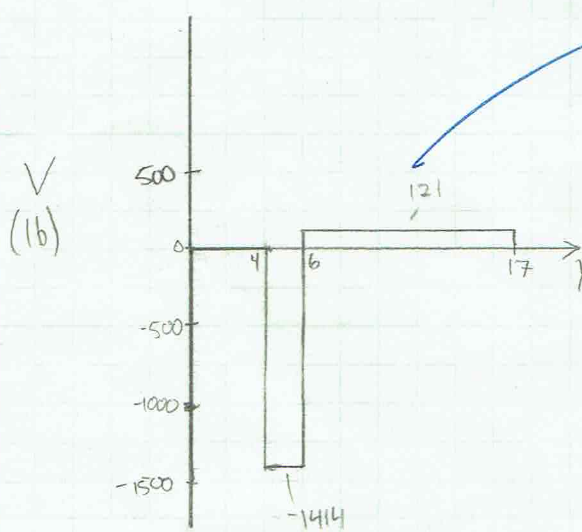
-1

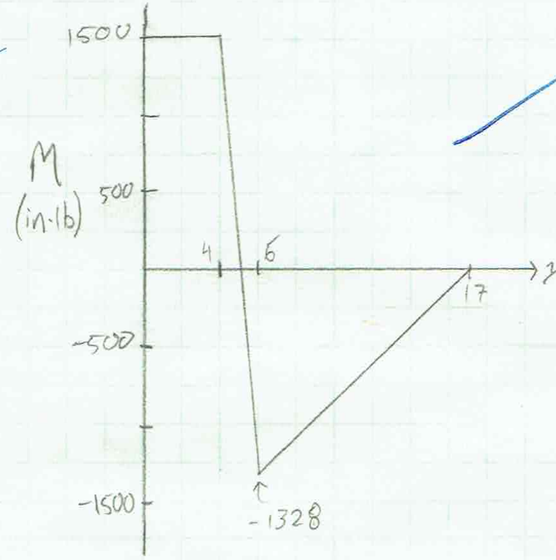
$+ M_0\langle x\rangle$

$V = -1810.6\cos(38.66)\langle y-4\rangle^0 + 2170.1\cos(\cancel{38.66}\,45)\langle y-6\rangle^0 - 154.6\cos(38.66)\langle y-17\rangle^0$

$M = 1500 - 1810.6\cos(38.66)\langle y-4\rangle^1 + 2170.1\cos(45)\langle y-6\rangle^1 - 154.6\cos(38.66)\langle y-17\rangle^1$

$$\boxed{\begin{aligned} V &= -1414\langle y-4\rangle^0 + 1535\langle y-6\rangle^0 - 121\langle y-17\rangle^0 \\ M &= 1500 - 1414\langle y-4\rangle^1 + 1535\langle y-6\rangle^1 - 121\langle y-17\rangle^1 \end{aligned}}$$





W
V
M
θ/EI
y/EI

4h

$$\boxed{\begin{gathered}\text{largest } V = 1414 \text{ lbs @ } 4<y<6 \\ \tau_{max} = 18 \text{ ksi}\end{gathered}}$$

$$\tau_{max} = \frac{VQ}{Ib} = \frac{(1414)(.00911)}{(.25)(0.00287)} = 17,953 \text{ psi}$$

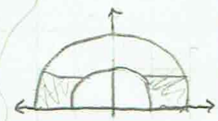
Wall thickness $\not< \sim\frac{1}{10} r_o$

$.125 \not< \sim\frac{1}{10} .25$

not thin walled

$$\tau = \frac{VQ}{Ib}$$

$\cancel{Q = \int_{y_1}^{c} y \, dA}$



max τ @ $y_1 = 0$

$$Q = Q_o - Q_i$$

where Q_o and Q_i are solid round beams

$$Q = \underbrace{\frac{2}{3}(r_o^2 - \cancel{y_1^2})^{\frac{3}{2}}}_{Q_o} - \underbrace{\frac{2}{3}(r_i^2 - \cancel{y_1^2})^{\frac{3}{2}}}_{Q_i}$$

$$Q = \frac{2}{3}\left[(r_o^2)^{3/2} - (r_i^2)^{3/2}\right]$$

$$= \frac{2}{3}\left[(.25^2)^{\frac{3}{2}} - (.125^2)^{\frac{3}{2}}\right]$$

$$Q = 0.00911 \text{ in}^3$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(.5^4 - .25^4)$$

$$I = 0.00287$$

4i

$$\boxed{\begin{gathered}\text{largest } M = 1500 \text{ in·lbs @ } y=6 \\ \sigma_{max} = 131 \text{ ksi}\end{gathered}}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1500(.25)}{.00287} \qquad c = r_o = .25$$

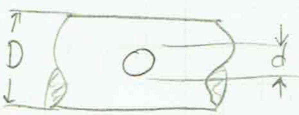
$$\sigma_{max} = 131 \text{ ksi}$$

4j

$$\boxed{\sigma_{max\,new} = 258 \text{ ksi}}$$

Figure C-7 $\quad K_t = 1.5899 - 0.6355 \log\left(\frac{d}{D}\right)$

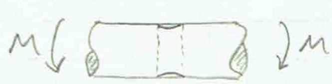
$$= 1.5899 - .6355 \log(.25)$$



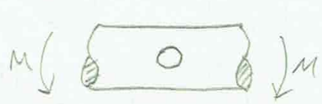
$$\frac{d}{D} = \frac{.125}{.5} = .25$$

$$\sigma_{max\,new} = K_t \sigma_{max} = 258 \text{ ksi} \qquad K_t = 1.973$$

- This is the wrong figure to use b/c the chart is for
  - solid bar
  - hole going "top - bottom"



But we have
  - hollow bar
  - hole going "sideways"



~~bI of circle $\rightarrow 2\sqrt{r^2 - y^2}\,\frac{\pi r^4}{4}$~~

~~@ y=0~~

~~$y_1 = 0 \quad = 2r\,\frac{\pi r^4}{4} = \frac{r^5 \pi}{2}$~~

~~bI of hollow tube $\rightarrow \frac{r_o^5 \pi}{2} - \frac{r_i^5 \pi}{2} = \frac{\pi}{2}(r_o^5 - r_i^5)$~~

~~$bI = \frac{\pi}{2}(.25^5 - .125^5) = .001486$~~

b = width @ center line = 2·thickness

$$b = 2(.125) = .25$$

(4k) considering the stress concentration, we need

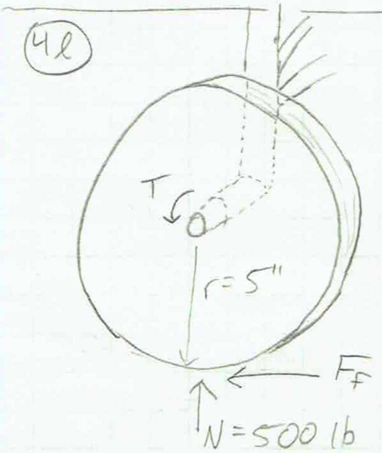
$\sigma_{ys} > 229$ ksi

Available candidates are:

alloy/tool steels:
- 4140 Q+T @ 400°F ($\sigma_{ys} = 238$) ksi
- 4340 Q+T @ 600°F ($\sigma_{ys} = 230$) ksi
- H-11 Q+T @ 1000°F ($\sigma_{ys} = 250$ ksi)
- L-2 Q+T @ 400°F ($\sigma_{ys} = 260$ ksi)
- L-6 Q+T @ 600°F ($\sigma_{ys} = 260$ ksi)
- S-1 Q+T @ 400°F ($\sigma_{ys} = 275$ ksi)
- S-5 Q+T @ 400°F ($\sigma_{ys} = 280$ ksi)

stainless steel alloy { SS type 440C Q+T @ 600°F ($\sigma_{ys} = 275$ ksi)

(4l)



$$F_F = \mu_K N = (0.25)(500) = 125 \text{ lb}$$

$$T = F \cdot d = (125_{\text{lb}})(5'') = \boxed{625 \text{ in} \cdot \text{lbs} = T}$$

$$\tau_{max} = \frac{Tr}{J} = \frac{(625)(.25)}{0.00575} \qquad J_{\odot} = \frac{\pi}{32}(D^4 - d^4)$$

$$= 0.00575$$

$$= 27162 \text{ psi}$$

$$G_{\text{carbon steel}} = 11.2 \times 10^6 \text{ psi}$$

↳ engineering toolbox.com

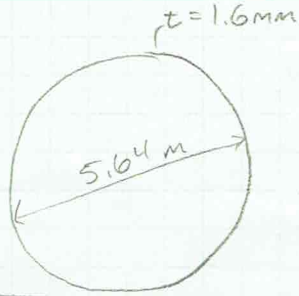
(4m) $\boxed{\tau_{max} = 27.2 \text{ ksi}}$

(4n) $$\phi = \frac{Tl}{JG} = \frac{(625)(3)}{(0.00575)(11.2 \times 10^6)} = 0.0291 \text{ rad}$$

$\boxed{\phi = 1.67°}$ angular deformation

5a

$t = 1.6\,mm$



Airbus a340 4-engine

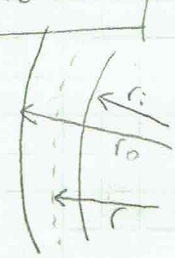
- Min skin thickness: 1.6 mm
  - "Development and Application of a New Design Tool for Aerospace Structures"
- Fuselage cross section based on a300
  - perfectly circular w/ 5.64 m dia
- 4-engine, a 380 uses 7040-T7451 al for fuselage
  - → "Aluminum alloy development for the airbus a380 – part 2"

$D = 5.64\,m \quad t = .0016\,m$

$r_o = 2.82\,m$

$r_i = 2.8184\,m$

$r = 2.8192\,m$



10

5b Model theory → stresses in closed ended, thin walled cylinder

$\sigma_t = \frac{pr}{t}$

$\sigma_r = 0$

$\sigma_a = \frac{pr}{2t}$

This model theory is chosen because the airplane acts like a pressure vessel when flying. The cabin must be sealed tight and withstand the pressure differential between inside + outside at cruising altitude. It is closed ended b/c the airplane acts like a closed-cylinder.

5c cruising altitude of a340 → 43,000 ft → $p_o = 16{,}235$ Pa

a340 "cabin altitude" → 7350 ft → $p_i = 77{,}152$ Pa

$p_o = 16{,}235$ Pa

$p_i = 77{,}152$ Pa

→ "Air Quality in Airplane Cabins and Similar Enclosed Spaces, Volume 4" pg 8

5d

$p = p_i - p_o = 77{,}152 - 16{,}235 = 60{,}917$ Pa

$\sigma_{radial} = 0$

$\sigma_{tangential\ max} = \frac{pr_o}{t} = \frac{(60{,}917)(2.82)}{.0016} = 107.4\,MPa$

$\sigma_{axial\ max} = \frac{pr_o}{2t} = \frac{(60{,}917)(2.82)}{2(.0016)} = 53.7\,MPa$

$\sigma_r = 0$

$\sigma_t = 107.4\,MPa$

$\sigma_a = 53.7\,MPa$

5e $\sigma_{ys} = 469\,MPa$ Type of Aluminum?

→ ASM Aerospace Specification Metals Inc.

$\frac{\sigma_{ys}}{\sigma_t} = \frac{469\,MPa}{107.4\,MPa} = 4.37$

$\boxed{SF = 4.37}$ - when considering a "Burst test" failure of the skin
- Doesn't consider stress concentrators or frame supports

$\sigma_{ys} = 4.4\,\sigma_t$

(6a)

OD: .675" ; avg thickness: .136"

R_o: .3375"

avg ID: .403"

avg R_i: .2015"

→ thickness $\overset{?}{<} \frac{1}{10}(\overset{r}{.2695})$

$.136 \not< .02695$

↳ thick walled

(6b) Theory: Thick walled, closed-ended pressure-vessel cylinder

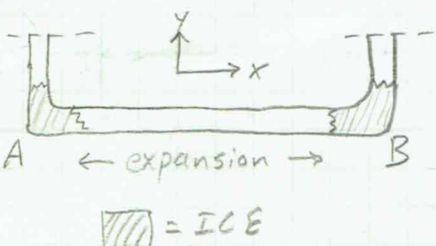
15

$$\hookrightarrow \sigma_t = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

- This was chosen because the expanding ice will create an ideally-uniform internal pressure on the pipe. • The pipe is thick walled b/c it's thickness is less than 1/10 it's radius. • The pipe is closed ended: suppose the water freezes at corners A and B. If water then freezes in the length between A and B, it will try to expand in the x-direction, causing an axial stress.



Note: Assume table PSI are absolute pressures

(6c) $\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)}$

$P_o = P_{atm} = 14.7$ Psi

PVC $\sigma_{ys} = 8000$ Psi

↳ engineering toolbox.com thermoplastics

$\sigma_r = \sigma_{cs} = 10,442 = \frac{p_i(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2}$

$r = r_i$

$- \frac{(.2015^2)(.3375^2)(p_i - 14.7)}{.2015^2(.3375^2 - .2015^2)}$

compressive strength

$\sigma_{cs} \cong 72$ MPa $= 10,442$ PSI

↳ PVC.org/en/p/pvc-strength

solve for

↳ $\boxed{p_{i_{max}} = 10442 \text{ psi}}$ @ compressive failure

max σ_r @ $r = r_i$

Using given max PSI, $p_i = 920$

$$\sigma_r = \frac{920(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2} - \frac{(.2015^2)(.3375^2)(920 - 14.7)}{.2015^2(.3375^2 - .2015^2)}$$

$$\boxed{\sigma_{r_{max}}(p_i = 920) = 920 \text{ psi}}$$

(6d) $\sigma_t = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2(r_o^2 - r_i^2)}$

$\sigma_{YS} = 8000$ psi
↳ engineering toolbox.com thermoplastics

$$8000 = \frac{p_i(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2} + \frac{.2015^2 .3375^2 (p_i - 14.7)}{.2015^2(.3375^2 - .2015^2)}$$

Max tensile tangential strength @ $r = r_i$

solve for p_i → $\boxed{p_i = 3817 \text{ psi}}$ @ yield

using given max PSI, $p_i = 920$

$$\sigma_t = \frac{920(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2} + \frac{(.2015^2)(.3375^2)(920 - 14.7)}{.2015^2(.3375^2 - .2015^2)}$$

$\boxed{\sigma_{t_{max}} (p_i = 920) = 1893.5 \text{ psi}}$

(6e) $\sigma_a = \sigma_{YS} = 8000 \text{ psi} = \frac{p_i(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2}$

↓

$\boxed{p_i = 14{,}485 \text{ psi}}$ @ yield

using given max $p_i = 920$ psi

$$\sigma_{a_{max}} = \frac{920(.2015^2) - 14.7(.3375^2)}{.3375^2 - .2015^2} = 486.7 \text{ psi}$$

$\boxed{\sigma_{a_{max}} (p_i = 920) = 486.7 \text{ psi}}$

(6f)

part ~~d~~ c) the max internal pressure was 10,442 psi, when considering the table's max p_i was 920 psi radial failure

This gives a SF of 11.35

part ~~e~~ d) the max internal pressure was 3817 psi, when considering the table's max p_i was 920 psi tangential failure (bursting)

This gives a SF of 4.15

part ~~f~~ e) The max p_i was 14,485 psi - when considering axial failure
The table's max p_i was 920 psi
↳ this would happen in rare cases, like explained in 6b

This gives a SF of 15.74

The most likely failure would be due to tangential forces, making the pipe burst open. The overall Safety Factor provided for 3/8" schedule 80 PVC pipe is 4.15. It was not calculated if expanding ice would apply a greater pressure than the recommended max pressure or pressure at failure. Note that, at freezing temp, the mechanical properties may change - changing the SF.