Show all work, state all assumptions. Figures not to scale. If you do not understand the problem statement, please ask for clarification. Write name on every page:

Heat Transfer
MCEN 3022 – Spring 2017
Exam 2

CU Honor Pledge

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

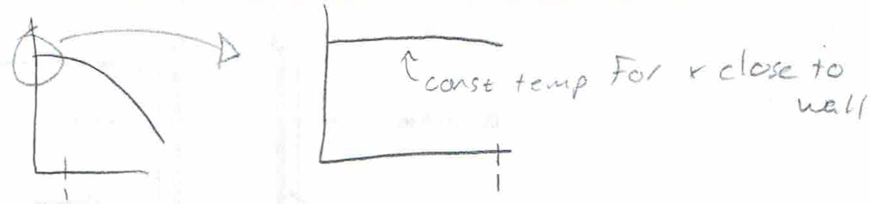
Student Name (printed): Ross Fischer

Signature: [illegible]

Important Note: The problem statements given below may contain extra information that is not needed for problem solution. Correspondingly, some information needed for problem solution may require values that are not provided in the problem statement, but are available in the textbook (e.g., density of water). Clearly state all assumptions used for problem solution. Show all work. Where requested, provide a brief explanation. Justify all answers quantitatively if possible.

1. (6 pts.) Consider the general case of a plane wall that is adiabatic on one side. Studying the region of the wall very close to the adiabatic surface would reveal that that temperature does not vary with

   A. Position
   B. Time
   C. A and B
   D. None of the above



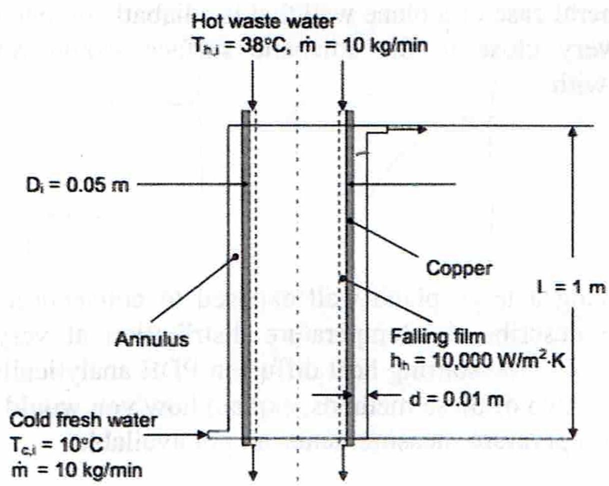
2. (12 pts.) You are studying a large plane wall exposed to convection on each side. Provide three methods to describe the temperature distribution at very early times. Assume you are unable to solve the starting heat diffusion PDE analytically but have the methods on the handout. For two of these methods, explain how you would check that the solution is valid. Assume temperature measurements are not available.

3. (12 pts.) You are designing a parallel flow concentric tube heat exchanger for which it is known that both that inner and outer (annular) fluids will undergo phase change throughout the heat exchanger. The fluid temperatures are therefore known. Your manager suggests finding the total heat rate using the log-mean temperature difference method or the effectiveness equation. Show that these methods will each fail to provide a meaningful result and provide an alternate (perhaps much simpler) method for finding the heat rate. Assume the total resistance R_{tot} is known.

Show all work, state all assumptions. Figures not to scale. If you do not understand the problem statement, please ask for clarification. Write name on every page:

4. (15 pts.) A temperature-measuring probe that is initially at a uniform temperature of 50 °C is suddenly submerged in an ice bath (0 °C). The thermal time constant, τ, for the probe is 4 seconds.
a) Find the time for 99% of the total temperature change (T_i - T_∞) to occur. Assume lumped capacitance.
b) How many multiples of τ does this time correspond to?

5. (28 pts.) President Foster has decided to conserve energy by installing counterflow concentric-tube heat exchangers in shower drains in new dorms. Hot wastewater from the shower enters the heat exchanger at 38 °C. Fresh water enters the outer, annular section of the heat exchanger at 10 °C. The heat exchanger consists of thin-walled copper drains of D = 50 mm. The outside surface enclosing the annular flow is insulated. The annular gap, d, (see figure below) is 10 mm. The mass flow rates for the wastewater and fresh water are each 10 kg/min. The length of the heat exchanger is 1 m. Because the wastewater creates a thin falling film, the heat transfer coefficient for the inside of the drain is h_h = 10,000 W/m^{2}K. The convection coefficient for the outer wall is h_c = 9050 W/m^{2}K (due to a helical structure guiding the flow around the pipe). Find the outlet temperature of the fresh water. Take the water properties to be k = 0.591W/mK, c_p = 4189 J/kgK throughout the heat exchanger.



~~5~~ 6. (27 pts.) A sphere is initially at a uniform temperature of 0 °C. The sphere is suddenly placed in boiling water at 100 °C with a convection coefficient h = 40,000 W/m^{2}K. The sphere is made of Molybdenum with ρ = 10,240 kg/m^3, c_p = 251 J/kgK, k = 138 W/mK, α = 53.7x10^{-6} m^2/s. The diameter of the sphere is 0.276 m. Find the time for the center of the sphere to reach a temperature of 99.5 °C.

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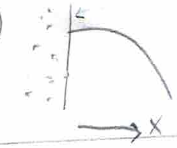
A.

① 

② 1- The infinite series solution (exact) $\frac{T-T_\infty}{T_i-T_\infty} = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\cos(\zeta_n x^*)$ +2

2- 1 term solution $(\theta^* = \theta_o^* \cos(\zeta_1 x^*))$ +1

3- Semi-infinite solid, case 3 $\left\{\theta^* = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right.$ +2

For the 1-term solution, this is valid for Fo #'s ≥ 0.2 +3

For the semi-infinite solid solution, this is valid so long as $Bi > 0.1$ and as $x \to \infty$, $T \to$ const. value (T_i) +1

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③ $q = UA \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$ phase changes: $\Delta T_1 = 0$, $\Delta T_2 = 0$

LMTD → $\boxed{q = 0 \text{ or undefined div by } 0}$ ✓

ε-NTU $\boxed{\begin{aligned} q &= \dot{m}_c C_{pc}(\Delta T_c) = 0 \\ q &= \dot{m}_H C_{pH}(\Delta T_H) = 0 \end{aligned}}$ ✓ $\varepsilon = \frac{q}{q_{max}} = \frac{0}{q_{max}} = 0 \to NTU = 0$

- Alternate method, since the Temps, and ΔTs, are known, use the eq → $q = \dot{m}_H C_{pH}(T_{HI} - T_{HO})$ ✓

→ Since R_{tot} is know, you may also use $\boxed{q = \frac{\Delta T}{R_{tot}}}$

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1) 6

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(4) - assume lumped capacitance for probe

$T_i = 50°C \qquad T_\infty = 0 \qquad \tau = 4 \text{ sec}$

(a) 99% of $(T_i - T_\infty)$ → when $T = 0.5°C$

$$\tau = \frac{\rho V c}{h A_s} = 4$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\left(\frac{hA_s}{\rho V c}\right)t\right) = \exp\left(-\frac{t}{\tau}\right)$$

$$\frac{0.5}{50} = .01 = \exp\left(-\frac{t}{4}\right)$$

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$$\ln(.01) = -\frac{t}{4}$$

$$\boxed{t = 18.42 \text{ sec}}$$

(b) $\frac{t}{\tau} = \frac{18.42}{4} = \boxed{4.61 \text{ multiples of } \tau}$

(5) $T_{Hi} = 38°C$ $T_{ci} = 10°C$

$T_{Ho} = ?$ $T_{co} = ?$

$$\boxed{T_{co} = 24.5°C}$$

thin walled → neglect conduction

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$L = 1$

$\dot{m}_c = \dot{m}_H = 10 \frac{kg}{min}$

$K_{water} = .591$ w/mK

$C_p = 4189$ J/kg k

$h_h = 10000 \frac{W}{m^2K}$

$h_c = 9050 \frac{W}{m^2K}$

$A_H = A_c = \pi D L = \pi(.05)$

$$C_H = C_c = \dot{m}_c C_p = (10 \tfrac{kg}{\cancel{min}})(4189 \tfrac{J}{kgk}) \tfrac{1 \cancel{min}}{60 sec} = 698 \tfrac{J}{ks} \text{ or } \tfrac{W}{K}$$

$C_r = 1$

$q_{max} = C_{min}(T_{Hi} - T_{ci}) = 698(38-10) = 19549$ W

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$$\varepsilon = \frac{\cancel{C_h}^{1}(T_{co} - T_{ci})}{\cancel{C_{min}}(\underbrace{T_{Hi} - T_{ci}}_{38-10})} = \frac{T_{co} - 10}{28} \rightarrow \varepsilon = \frac{T_{co}}{28} - .357$$

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c(T_{co} - T_{ci})}{19549}$$

thin walled

$$\frac{1}{UA_o} = \frac{1}{(hA)_c} + \cancel{R_w} + \frac{1}{(hA)_h} = \frac{1}{(9050)}$$

$$\frac{1}{U_o} = \frac{1}{h_c} + \frac{1}{h_H} = \frac{1}{9050} + \frac{1}{10{,}000}$$

might need $\frac{L}{KA}$ from water conduction?

$U_o = 4751 \frac{W}{m^2K}$

$$NTU = \frac{UA}{C_{min}} = \frac{(4751)(\pi(.05))}{698} = 1.0691$$

"thin wall"

$L \rightarrow 0$

eq 11.29a, $C_r = 1$, $\varepsilon = \frac{NTU}{1 + NTU} = \frac{1.0691}{2.0691} = 0.5167$

$$\varepsilon = \frac{T_{co}}{28} - .357 = .5167$$

$T_{co} = 24.5°C$

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(6) $T_i = 0°C$ $\quad T_\infty = 100°C$ $\quad h = 40{,}000\ \frac{W}{m^2K}$

$\rho = 10240\ \frac{kg}{m^3}$ $\quad c_p = 251\ \frac{J}{kgK}$ $\quad k = 138\ \frac{W}{mK}$

$\alpha = \frac{k}{\rho c} = 53.7\times10^{-6}\ \frac{m^2}{s}$

$D = .276\ m \rightarrow \frac{r_o}{3} = .046 \rightarrow r_o = .138$

check for L.C. $\quad Bi = \frac{hL_c}{k} = \frac{(40{,}000)(.046)}{138} = 13.3 \rightarrow$ Nope L.C.

$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{99.5-100}{0-100} = .005 = C_1 \exp(-\zeta_1^2 F_o)$

$Bi = \frac{hL_c}{k} = \frac{(40{,}000)(\overset{r_o}{.138})}{138} = 40 \rightarrow$ table 5.1 $\zeta_1 = 3.0632,\ \zeta_1^2 = 9.3832$

$\rightarrow C_1 = 1.9942$

$.005 = (1.9942)\exp(-9.3832\ F_o)$

$-5.9885 = -9.3832\ F_o$

$F_o = 0.6382 = \frac{\alpha t}{r_o^2} = \frac{(53.7\times10^{-6})t}{(.138)^2}$

$\boxed{t = 226.3\ sec}$ $= 3.77$ min

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