Show all work, state all assumptions. All answers in order on separate paper. Figures not to scale.
Write name on every page:

Mechanical Engineering Fluid Mechanics
MCEN 3021 – Fall 2016
Exam 2

CU Honor Pledge
On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

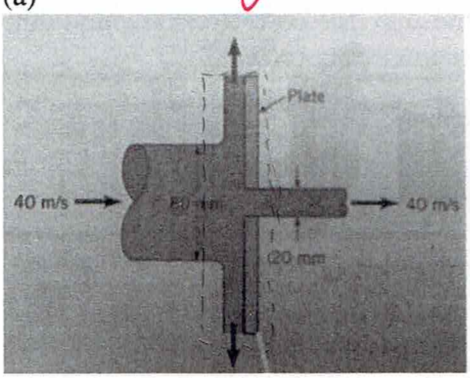
Student Name (printed): Ross Fischer

Signature: Ross Fischer

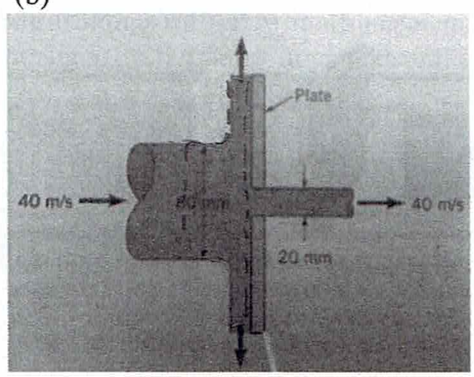
Important Note: The problem statements given below may contain extra information that is not needed for problem solution. Correspondingly, some information needed for problem solution may require values that are not provided in the problem statement, but are available in the textbook (e.g., density of water). Clearly state all assumptions used for problem solution. Show all work. Where requested, provide a brief explanation.

1. (12 pts) Consider the free jets of water hitting circular plates shown below.
Draw below on exam for (a), (b).
   (a) Draw the CV to determine the anchoring force (x-comp.) for the plate.
   (b) Draw the CV to determine the reactive force (x-comp.) on the water jet.
   (c) What is the direction of the x-component of the reactive force? Reactive force will act to the left
   (d) Could gage pressure be used in case (a)? Explain.
   (e) Could gage pressure be used in case (b)? Explain.

(a) ✓



(b) ✓



2. (8 pts) (a) Is it possible for an incompressible flow to experience both positive local acceleration and negative convective acceleration in a region of flow? If so, provide an example and explain.

(b) Is it possible for a steady-state flow to experience both positive local acceleration and negative convective acceleration in a region of flow? If so, provide an example and explain.

Show all work, state all assumptions. All answers in order on separate paper. Figures not to scale.
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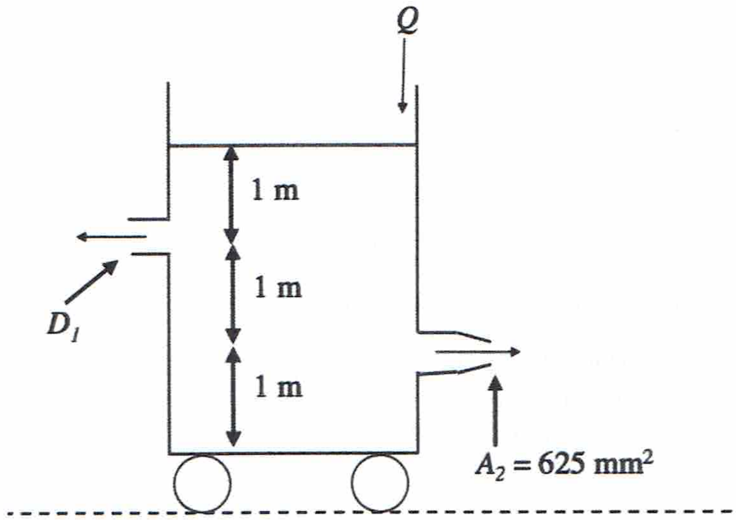
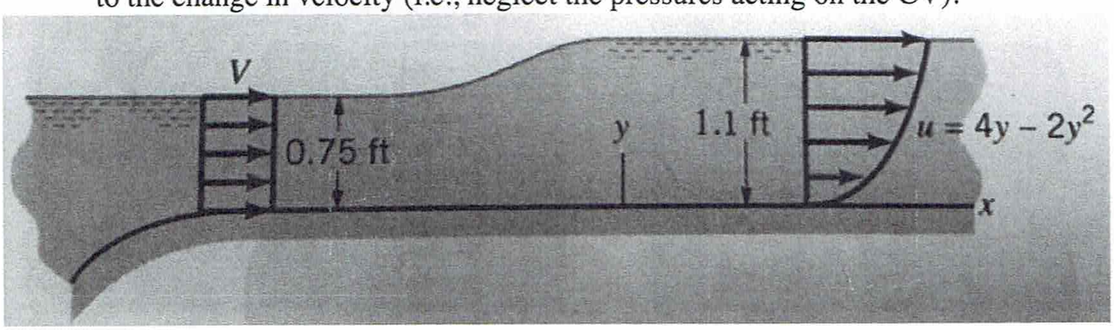
3. (20 pts) Water is added to the tank shown below at a rate Q to maintain a constant water level. The tank is on frictionless wheels. Determine the diameter ~~D_1~~ D_1 for the tank to not move. Neglect viscous losses.



4. (25 pts) Seawater (steadily) enters the 3-foot-wide channel shown below at uniform velocity V. Downstream of the entrance the velocity distribution is $u=4y-2y^2$ ft/s where y is the ocean height in feet. Note the y-origin is at the ocean floor. (Units might appear odd when examining outlet.)

(a) Determine the entrance velocity V.

(b) Determine the x-component of the force (with direction) exerted by the water on the ocean floor (over the region from the entrance to the downstream section) due to the change in velocity (i.e., neglect the pressures acting on the CV).



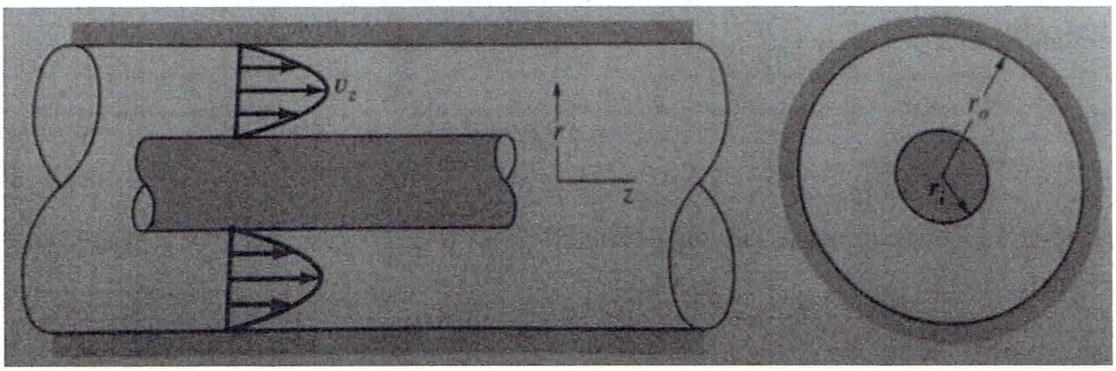
Show all work, state all assumptions. All answers <u>in order</u> on separate paper. Figures not to scale.
Write name on every page:

5. (20 pts) Water enters in a (regular) circular pipe of diameter 14 cm with uniform velocity at steady state. The flow exits through the annular region shown below with an outer radius r_o of 7 cm and inner radius r_i of 3 cm. The pressure drop $\frac{\partial P}{\partial z}$ is – 0.2 Pa/m and is a constant. The oulet velocity in the annular region is given by,

$$V_z = \frac{1}{4\mu}\left(\frac{\partial P}{\partial z}\right)\left(r^2 - r_o^2 + \frac{r_i^2 - r_o^2}{\ln(r_o/r_i)}\ln(r/r_i)\right).$$

Determine the inlet velocity. Note the following integrals where k is a constant. *You may* *not perform the integral with your calculator*. Introduce $B = \frac{1}{4\mu}\left(\frac{\partial P}{\partial z}\right)$ and $E = \frac{r_i^2 - r_o^2}{\ln(r_o/r_i)}$.

$\int \ln(kx)dx = x(\ln(kx)-1)$, $\int x\ln(kx)dx = \frac{x^2}{4}(2\ln(kx)-1)$, $\int \ln(kx^2)dx = x(\ln(kx^2)-2)$



6. (15 pts) Hint: consider what you'd do if this question were given in Cartesian coordinates. Consider a 3-dimensional flow in spherical coordinates (that has a fluid source at the origin). You are told that the flow is incompressible and the velocity distribution is given by,

$$V_r = K_1\left(\frac{r^2}{R^2}\right) + K_2\left(\frac{r}{R}\right) + V_0$$

$$V_\phi = -2\sin\theta\left(\frac{3K_2 r\phi}{2R} + \phi V_0\right)$$

$$V_\theta = -4K_1 r^2/R^2,$$

where K_1, K_2, V_0, and R are constants. Would you agree that this flow appears to be incompressible? The differential mass balance in spherical coordinates is given below.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\rho V_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\rho V_\phi\right) = 0$$

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1

(1) c) Reactive force will act to the left

d) Gage Pressure could be used.
Patm is acting all around the CV, and will cancel itself out. All that's left is Gage pressure. Or, the anchoring force is independent of atmospheric pressure.

e) Gage Pressure could not be used, because the Reactive force depends on atmospheric pressure. When looking at the Force between a Fluid-solid interface, it will depend on all pressures surrounding it, including atmospheric.

$\frac{12}{12}$

Great Work!!!

$\frac{98}{100}$

2) It is possible. An incompressible Flow

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a) can experience an increase in local accel because it's not steady state, and it can experience a negative convective accel. (If it turns in a pipe, for example, it will have a negative acceleration wrt the x or y coordinate.)

b) A steady state flow means the vel at a given point doesn't change with time, $\frac{\partial V}{\partial t} = 0$. This means it can't experience a positive local acceleration at all.

$\frac{8}{8}$

#3

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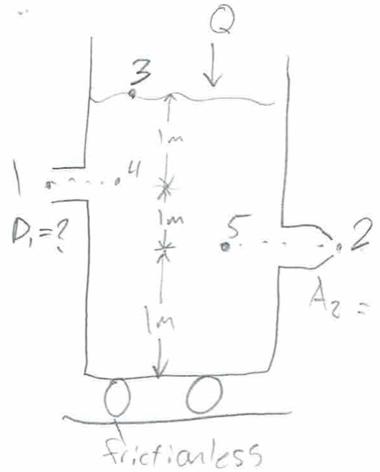

Y X

$A_2 = 625\,mm^2 \cdot \frac{m^2}{1000^2\,mm^2} = 6.25\times10^{-4}\,m^2$

20/20

Det D_1 for tank to not move. Neglect viscous losses.

Pressure - height $\underline{3\to4}$

$P_4 = \cancel{P_3}^{atm} + \rho g h = (1000)(9.81)(1)\frac{Kg}{m^3}\cdot\frac{m}{s^2}\cdot m$

gage

$P_4 = 9810$ Pa gage

$3\to5$

$P_5 = \cancel{P_3}^{atm} + \rho g h = (1000)(9.81)(2)$

$P_5 = 19{,}620$ Pa gage

Bern $4\to1$

$P_4 + \frac{1}{2}\rho \cancel{V_4}^2 + \rho g \cancel{z_4} = \cancel{P_1} + \frac{1}{2}\rho V_1^2 + \rho g \cancel{z_1}$

$V_4 = 0$ same height atm same height

$9810 = \frac{1}{2}\rho V_1^2 \rightarrow V_1 = \sqrt{\frac{2\cdot P_4}{\rho}}$

$V_1 = 4.429\ \frac{m}{s}$

$5\to2$

$P_5 + \frac{1}{2}\rho \cancel{V_5}^2 + \rho g \cancel{z_5} = \cancel{P_2} + \frac{1}{2}\rho V_2^2 + g\rho \cancel{z_2}$

$V_5 = 0$ same z atm same z

$P_5 = \frac{1}{2}\rho V_2^2 \rightarrow V_2 = \sqrt{\frac{2 P_5}{\rho}}$

$V_2 = 6.264\ \frac{m}{s}$

momentum balance

x direction

$\cancel{\frac{\partial}{\partial t}\left(\int_{CV} \rho \bar{V}\, d\forall\right)} + \int_{CS} \bar{V}\rho(\bar{v}\cdot\hat{n})\, dA = \Sigma F_x = 0$

system not moving or CV not changing

$\pi r^2 \quad \frac{\pi}{4} d^2$

$-u_1 \cancel{\rho} V_1 A_1 + u_2 \cancel{\rho} V_2 A_2 = 0$

$A_1 = \frac{\pi}{4} D_1^2$

$(6.264)^2(6.25\times10^{-4})\frac{\cancel{m^2}}{\cancel{s^2}} m^2 = (4.429)^2 \frac{\pi}{4} D_1^2 \frac{\cancel{m}}{\cancel{s^2}}$

$D_1 = .0398\,m$

$\boxed{D_1 = 39.9\ mm}$

4

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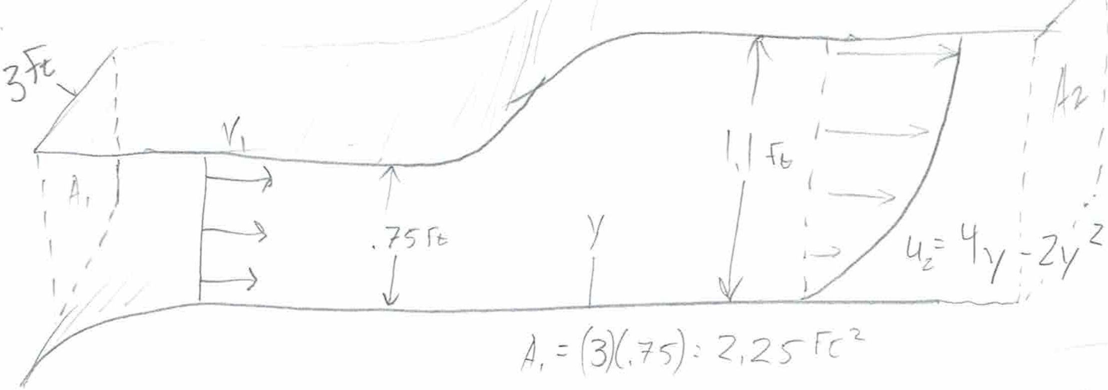
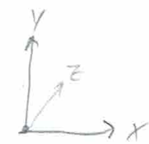
4





$A_1 = (3)(.75) = 2.25\ Ft^2$

$A_2 = 3.3\ Ft^2$

- assume incomp.
- steady state

Seawater $\rho = \cancel{1030\ \frac{kg}{m^3}}$
$= 1.99\ \frac{slugs}{Ft^3}$

a) Det entrance Vel V_1
mass balance

$\dot{m}_{in} = \dot{m}_{out}$

$$\int \cancel{\rho} (\bar{v}_1 \cdot \bar{n})\, dA_1 = \int \cancel{\rho} (\bar{v}_2 \cdot \bar{n})\, dA_2$$

u_2^2

$$V_1 A_1 = \int_0^{1.1} \int_0^3 (4y - 2y^2)\, dz\, dy$$

$$V_1 (2.25)\ Ft^2 = 3 \int_0^{1.1} 4y - 2y^2\, dy$$

$$V_1 (2.25) = 3\left[2y^2 - \frac{2}{3}y^3 \Big|_0^{1.1}\right] = 3\left[2.42 - .887\right] = 4.598 \quad \therefore \boxed{V_1 = 2.044\ \frac{Ft}{s}}$$

b) Det x component of force (w/ direction) exerted by water on floor due to ΔV (momentum balance)

$$\cancel{\frac{\partial}{\partial t} \int_{CV} (\rho \bar{v}\, d\forall)} + \int_{CS} \bar{v} \rho (\bar{v} \cdot \bar{n})\, dA = \sum F =$$

CV not changing

- neglect pressures on CV

$$-V_1^2 \rho A_1 + 3\rho \int_0^{1.1} u_2^2\, dy = F_{floor}$$

$$-(4.177)(1.99)(2.25) + 3(1.99)\int_0^{1.1} 16y^2 - 16y^3 + 4y^4\, dy = F_{floor}$$

$$-18.71 + 5.97\left[\frac{16}{3}y^3 - 4y^4 + \frac{4}{5}y^5\right]_0^{1.1} = F_{floor}$$

$u_2^2 = (4y - 2y^2)(4y - 2y^2)$
$= 16y^2 - 8y^3 - 8y^3 + 4y^4$

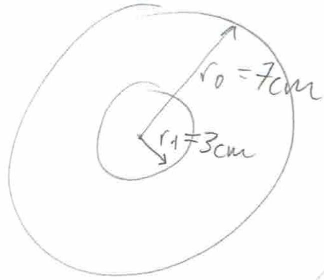
$$-18.71 + 5.97(2.53) = \boxed{F_{floor} = -3.6\ lb}$$
$+15.11$

Force acts to the left

F of floor on water

$\frac{24}{25}$

#5



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pressure drop, constant

$\frac{\partial P}{\partial z} = -.2 \frac{Pa}{m}$

- Steady state
- incomp
  - $\dot{m}_{in} = \dot{m}_{out}$

$\mu = 1.12 \times 10^{-3} \frac{N \cdot s}{m^2}$

uniform V_1

Outlet velocity $V_z = \frac{1}{4\mu}\left(\frac{\partial P}{\partial z}\right)\left(r^2 - r_0^2 + \frac{r_i^2 - r_0^2}{\ln(r_0/r_i)}\ln(r/r_i)\right)$

$B = \frac{1}{4\mu}\left(\frac{\partial P}{\partial z}\right)$

$E = \frac{r_i^2 - r_0^2}{\ln(r_0/r_i)}$

Det inlet velocity.

$\int \cancel{\rho}(\bar{v}_1 \cdot \bar{n})\, dA_1 \pm \int \cancel{\rho}(\bar{v}_2 \cdot \bar{n})\, dA_2 = 0$

ρ cancels

$-V_1 A_1 = 2\pi \int_{.03}^{.07} V_z\, r\, dr = 2\pi \int_{.03}^{.07} \frac{-.2}{4\mu}\left(r^3 - r r_0^2 - r\frac{.004}{\ln\left(\frac{.07}{.03}\right)}\ln\left(\frac{r}{.03}\right)\right) dr$

$+V_1 A_1 = +2\pi\left(\frac{.2}{4\mu}\right)\int_{.03}^{.07} r^3 - r r_0^2 - r\frac{.004}{\ln\left(\frac{.07}{.03}\right)}\ln\left(\frac{r}{.03}\right) dr$

$\int x \ln(kx)\, dx = \frac{x^2}{4}\left(2\ln(kx) - 1\right)$

$V_1 A_1 = 2\pi\left(\frac{-.2}{4\mu}\right)\left[\frac{r^4}{4} - \frac{r^2 r_0^2}{2} - \frac{r^2}{4}\frac{.004}{\ln\left(\frac{.07}{.03}\right)}\left(2\ln\left(\frac{r}{.03}\right) - 1\right)\right]_{.03}^{.07}$

$A_1 = \pi(.07)^2 =$

$R_1 = .07\,m$

$=\mu?$

$V_1(\pi \cdot .07^2) = 2\pi\left(\frac{-.2}{4 \cdot 2.34 \times 10^{-5}}\right)\left[-1 \times 10^{-5} - \left(-9.4 \times 10^{-7}\right)\right]$

-1

$V_1(.07^2) = -89.29\left[-9.06 \times 10^{-6}\right]$

$\frac{19}{20}$

used $\mu = 1.12 \times 10^{-3}$ ok

$V_1 = .165 \frac{m}{s}$

#6.) consider 3d flow in spherical coord's (fluid source @ origin)

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- flow incomp
- vel dist...

$$V_r = K_1\left(\frac{r^2}{R^2}\right) + K_2\left(\frac{r}{R}\right) + V_0$$

$$V_\phi = -2\sin(\theta)\left(\frac{3K_2 r\phi}{2R} + \phi V_0\right)$$

$$V_\theta = \frac{-4K_1 r^2}{R^2}$$

- K_1, K_2, V_0, R are consts.
- WO

- Does this flow appear to be incomp?

spherical continuity...

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho V_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho V_\phi) = 0$$

- If flow were incomp, we can cross out the above terms $\frac{\partial\rho}{\partial t}$, and ρ and the eq. should still be satisfied.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_1\left(\frac{r^2}{R^2}\right) + r^2 K_2\left(\frac{r}{R}\right) + r^2 V_0\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\frac{4K_1 r^2}{R^2}\sin(\theta)\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(-2\sin\theta\left(\frac{3K_2 r\phi}{2R} + \phi V_0\right)\right) \stackrel{?}{=} 0$$

$$\frac{1}{\cancel{r^2}}\left(\frac{4K_1 r^{\cancel{3}}}{R^2} + \frac{3\cancel{r^2}K_2}{R} + \frac{2\cancel{r}V_0}{r}\right) + \frac{1}{\cancel{r}\sin\theta}\left(-\frac{4K_1 r^{\cancel{2}}}{R^2}\cos(\theta)\right) + \frac{1}{\cancel{r}\cancel{\sin(\theta)}}\left(-2\cancel{\sin(\theta)}\left(\frac{3K_2\cancel{r}}{2R} + \frac{V_0}{r}\right)\right) \stackrel{?}{=} 0$$

$$\frac{4K_1 r}{R^2} + \cancel{\frac{3K_2}{R}} + \cancel{\frac{2V_0}{r}} - \frac{4K_1 r}{R^2}\frac{\cos\theta}{\sin\theta} - \cancel{\frac{3\phi K_2}{2R}} - \cancel{\frac{2V_0}{r}} \stackrel{?}{=} 0$$

$$\frac{4K_1 r}{R^2} - \frac{4K_1 r}{R^2}\frac{\cos\theta}{\sin\theta} = 0$$

$$\cancel{\frac{4K_1 r}{R^2}}\left(1 - \frac{\cos\theta}{\sin\theta}\right) = 0 \quad \rightarrow \quad \frac{\cos\theta}{\sin\theta} \neq 1$$

13/15

The flow does not seem to be incompressible.