Fourier transformations are a representation of a function by a combination of sinusoidal functions. Fourier transformation detect the frequencies that compose the original function. Each frequency has corresponding amplitudes, A_k and B_k. The summation of these sinusoidal functions can be used to represent the function as shown in equation 1.1, where L=(b-a)/2.

$$f(x) = \sum_{k=0}^{\infty} \left[A_k \sin\left(\frac{\pi k x}{L}\right) + B_k \cos\left(\frac{\pi k x}{L}\right) \right] \qquad \text{Eq. 1.1}$$

The coefficients for each term k is given by equations 1.2 – 1.4.

$$A_k = \frac{1}{L} \int_a^b f(x) \sin\left(\frac{\pi k x}{L}\right) dx \qquad \text{Eq. 1.2}$$

$$B_k = \frac{1}{L} \int_a^b f(x) \cos\left(\frac{\pi k x}{L}\right) dx \qquad \text{Eq. 1.3}$$

$$B_0 = \frac{1}{2L} \int_a^b f(x)\, dx \qquad \text{Eq. 1.4}$$

Problem 1

We are asked to solve problem 7.6 by obtaining the coefficients for a Fourier series using k=10 terms, and k=50 terms. The function we are analyzing is a triangle wave with a period of 2.

We found the coefficients for a general infinite Fourier series using equation 1.1 by using the coefficients found from equation 1.2 – 1.4.

The series using 10 and 50 terms was plotted against the original triangle wave and shown in figure 1. The amplitude of the coefficients were also plotted against the frequency of the corresponding sin and cos terms also shown in figure 1. The frequencies were found by considering the function *sin(w_ot),* where $w_o = 2\pi f$. In our case, $w_o = 2\pi f = \pi k/L$, so $f = k/(2L)$.

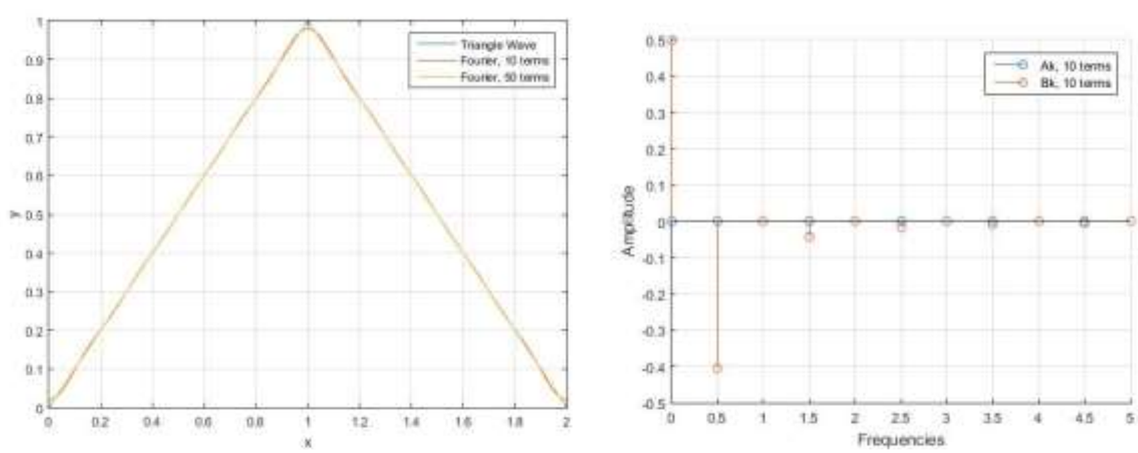

Figure 1. Fourier series approximation of a triangle wave, and corresponding coefficients plotted against their frequencies.

Both the 10 term and 50 term approximation was very similar to the actual function, and deviated the most at the peak of the function.

In the case of N=10 terms, only the first four terms were used to produce the plot in figure 2. This should be the same plot if N=3

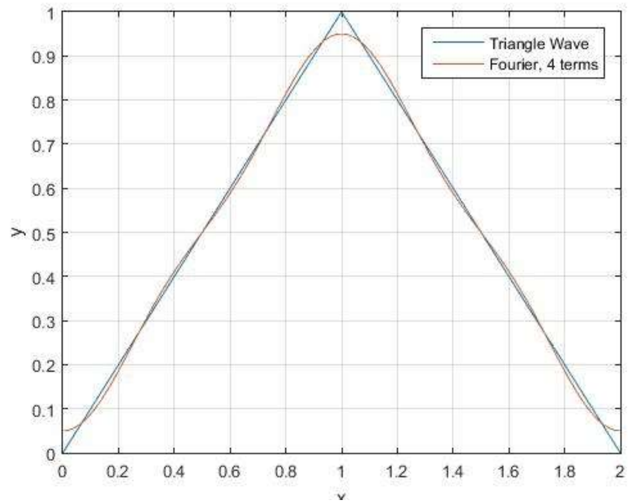

Figure 2. General Infinite Fourier Series on triangle wave using four terms.

Problem 2

Similar to the general infinite series, a Fourier series can be produced for a discrete set of data that approximates it by finding a combination of sinusoidal function with different frequencies and amplitudes over the interval [0, τ]. This approximation is shown in equation 2.1, where the number of data points is equal to *2N+1*, and $t_j = \tau\,(j-1)/(2N)$. The coefficients are given by equations 2.2 – 2.6

$$f(t_j) = \sum_{k=0}^{N} \left[A_k \sin\left(\frac{2\pi k t_j}{\tau}\right) + B_k \cos\left(\frac{2\pi k t_j}{\tau}\right) \right] \qquad \text{Eq. 2.1}$$

$$A_0 = A_N = 0 \qquad \text{Eq. 2.2}$$

$$A_k = \frac{1}{N} \sum_{j=1}^{2N} f(t_j) \sin\left(\frac{2\pi k t_j}{\tau}\right) \qquad for\ k = [1, N-1] \qquad \text{Eq. 2.3}$$

$$B_0 = \frac{1}{2N} \sum_{j=1}^{2N} f(t_j) \qquad \text{Eq. 2.4}$$

$$B_N = \frac{1}{2N} \sum_{j=1}^{2N} f(t_j) \cos\left(\frac{2\pi N t_j}{\tau}\right) \qquad \text{Eq. 2.5}$$

$$B_k = \frac{1}{N} \sum_{j=1}^{2N} f(t_j) \cos\left(\frac{2\pi k t_j}{\tau}\right) \qquad for\ k = [1, N-1] \qquad \text{Eq. 2.6}$$

We are given three sets of sample data and asked to perform a Discrete Fourier Transformation (DFT) on them. The coefficients for A_k and B_k were plotted against frequency. The frequency was found using the same method as problem 1. For a DFT, $f = k/\tau$.

The first data set, as shown in figure 3, appears to be a sin function with an amplitude of 5 with a period of 20. Our only coefficient, as seen in figure 3 to the right, is A_k=5, when *k=1*. Plugging these values into equation 2.1 provides the equation $f(t_j) = 5*sin(2*\pi*.05*t_j)$, which confirms our observation. The Nyquist criterion provides a minimum sampling rate of .1 Hz.

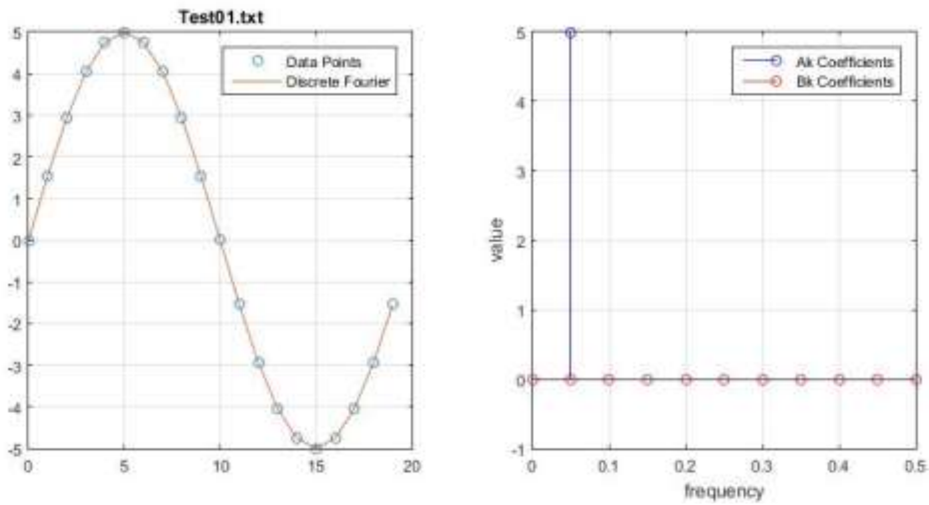

Figure 3. DFT of Test01.txt data plotted against the data itself, and the coefficients plotted against their frequencies.

Similarly, the second data set appeared to be just a cos function with an amplitude of 5 with a period of 20 as shown in figure 4. Our only coefficient, $B_k = 5$ when *k=1*, provides the equation $f(t_j) = 5*cos(2*\pi*.05*t_j)$, which confirms our observations. The Nyquist criterion provides a minimum sampling rate of .1 Hz.

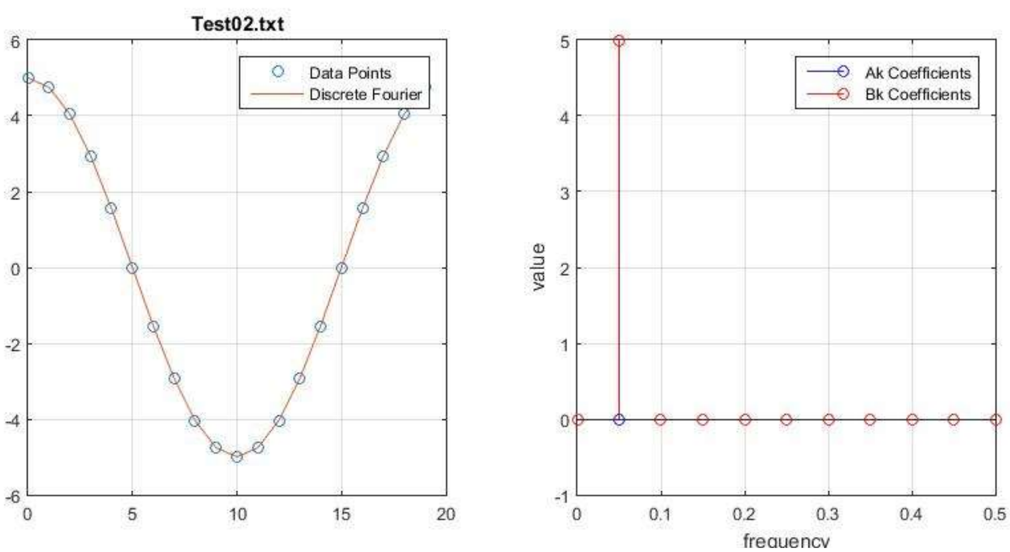

Figure 4. DFT of Test02.txt data plotted against the data itself, and the coefficients plotted against their frequencies.

Our third set of data looks sinusoidal at first and similar to the first data set, however the next quarter of a period of the wave was included in the data set. The function can still be periodic if repeated over and over, but it would not be sinusoidal, as shown in figure 5.

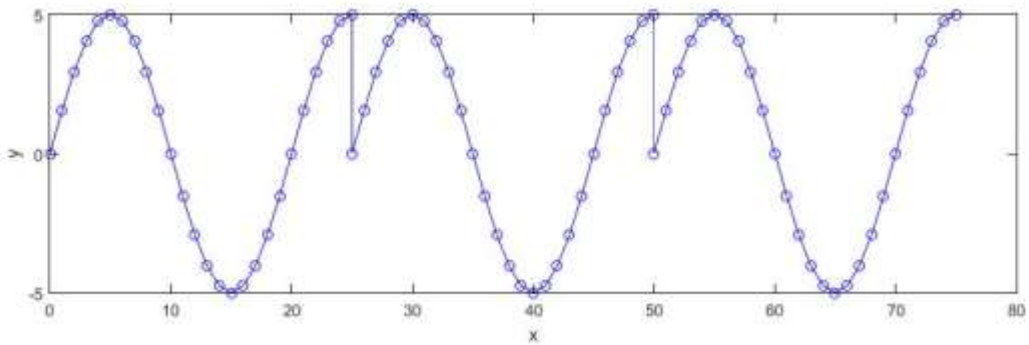

Figure 5. Repeated data from Test03.txt.

Since the DFT equations assume the data is periodic, it is trying to approximate the function shown in figure 5. The DFT was still performed and the results are shown in figure 6. For this function, the Nyquist frequency would be 1.04 Hz.

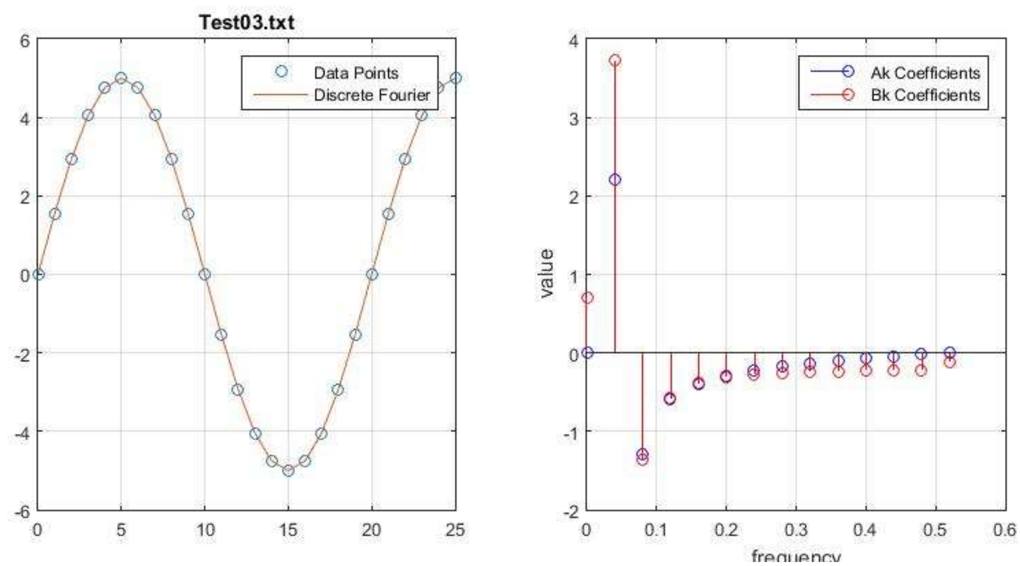

Figure 6. DFT of Test02.txt data plotted against the data itself, and the coefficients plotted against their frequencies.

If the data was meant to be sinusoidal, the DFT would have to be performed on the period between *x=0* and *x=20.*

Problem 3

We were provided with a data set which appears to represent a heart beat, as shown in figure 7.

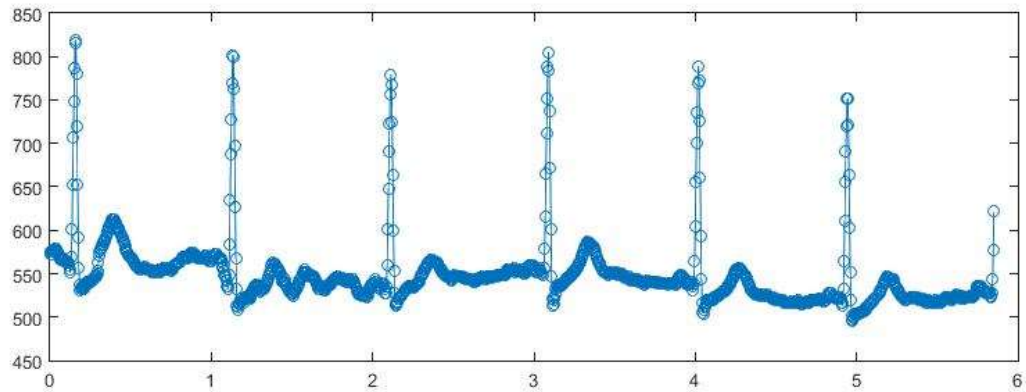

Figure 7. Data from Cardio.txt

To perform a DFT on the data, we must pick a set of data points representing one period. I chose points 33 through 282 to represent a period. After performing the DFT, figure 8 was produced. There were too many frequencies to plot the coefficients for, so those close to zero are not shown. The highest coefficient was 571 at a frequency of 0 Hz. When considering those values in relation to equation 2.1, they correspond to a constant value of 571, which provide a "baseline" for the rest of the summation to act upon.

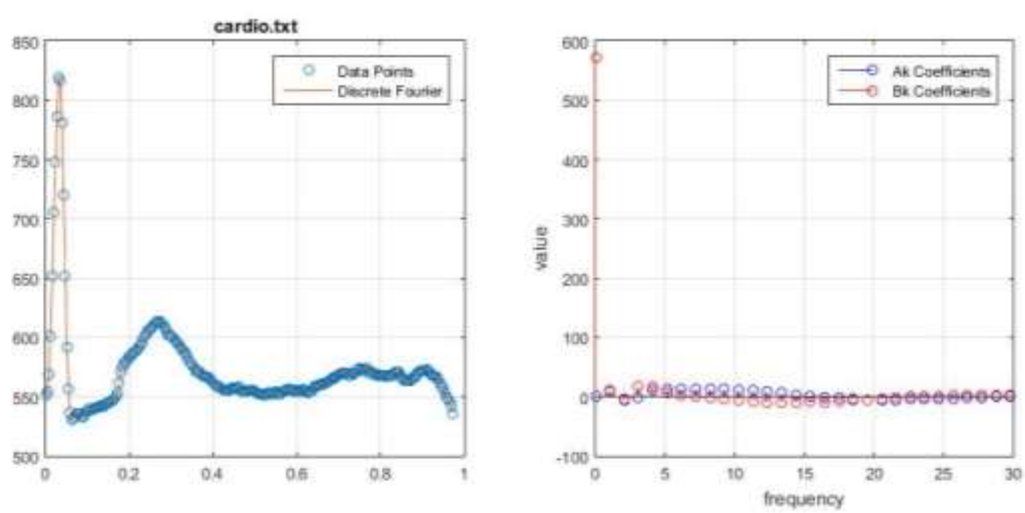

Figure 8. One period of cardio.txt and the DFT, and the coefficients plotted against their frequencies.