Ross Fischer	**Aerospace Environment**
Fall 2023	**ASEN 5335**
Module 3 / Homework 3	

Exercise 1, (20) Plotting the Ionosphere:

1. Use the International Reference Ionosphere (IRI-2007) hosted by the CCMC (Click on "Instant Run" at the top left). Run the model to create an output file changing only the following parameters:
Year, Month, Day, Time: 2006, Dec 21, Noon
Time: Local
Lat, Lon: 40N, -105E (255W)
Altitude range: 0 to 1000 km
Step Size: 5 km
DO NOT submit the figure that is generated on the website; download the data ("View Raw Output" at the top; copy and paste into a text file) and then make your own plots

Part 1)i)

Plot altitude (on the y-axis) vs. number density (log scale, on the x-axis) for electrons, O+, H+, O2+, NO+, and He+. Use your plot to answer the following questions:

To find each constituent's number density, take it's composition percentage multiplied by the total number density. Note the model output had some constituent percentages starting at negative one, which I set to zero.

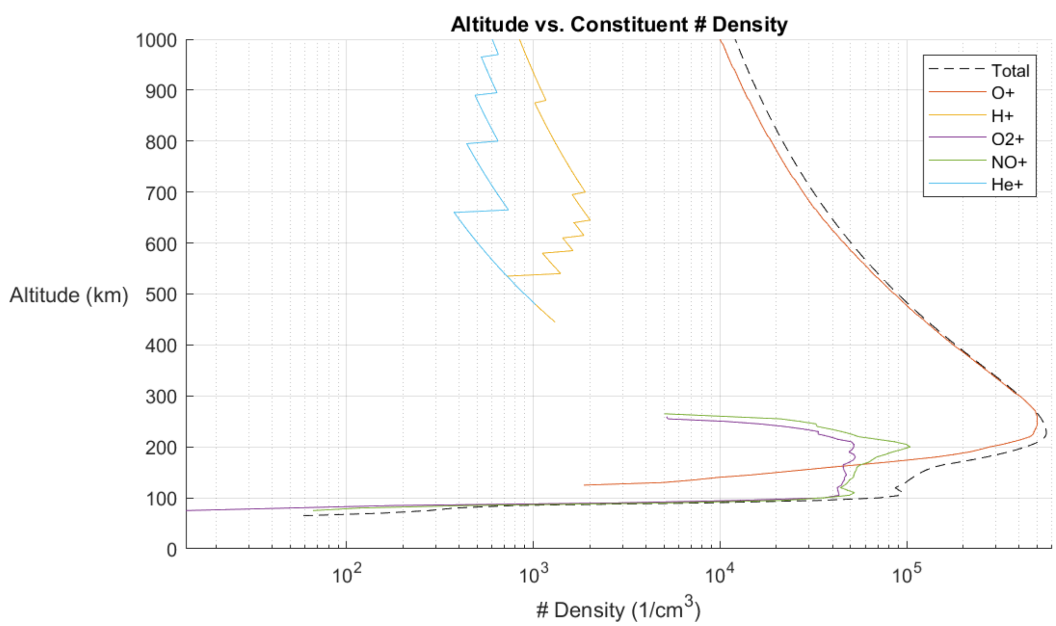

Figure 1: Part 1i) Number Density for requested constituents

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Part 1)ii)

Below 500 km, over what altitude range does NO+ dominate? At what range of altitudes does O2+ contribute more than 20% of the total?

- Below 500 km, NO+ dominates between 70 and 165 km.
- The altitude range where O2+ contributes $\geq$ 20% is between 85 and 180 km.

Part 1)iii)

What is the total electron content (TEC)? What is the TEC in the topside ionosphere (region above the peak electron density)? What is the TEC in the bottom side ionosphere (region below the peak electron density)? Report your answers in TEC units (TECU).

The total electron current is the total # of electrons in a square-unit area column between a transmitter and receiver. The model output provides TEC as 13 $TECU = 13 \cdot 10^{16}\ m^{-2}$, for every altitude, but TEC is actually dependent on the path (i.e. altitude, in this case).

By integrating for each 5km step, we can find the TEC from ground to 1000km...

$$Total\ Electron\ Current\ (0\ to\ 1000km) = 12.51\ TECU$$

This must mean the model uses a different approach to calculating TEC.

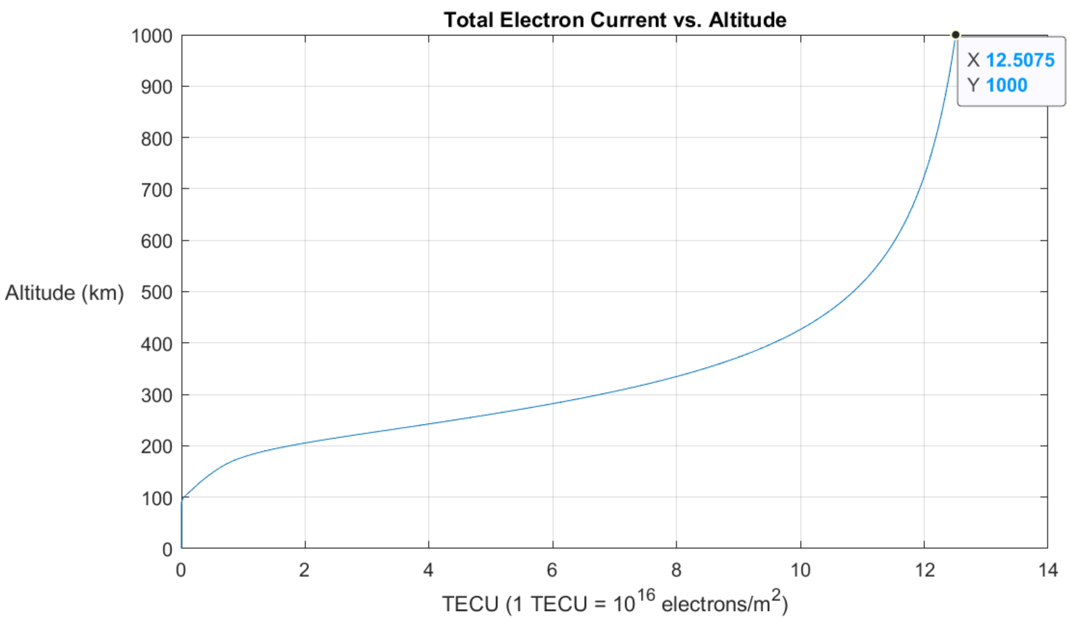

Figure 2: Part 1)iii) Total electron current vs. Altitude

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Exercise 2, (10) Peak Ionization:

In a Chapman layer, the peak ionization production rate, qmax, is related to the electron density $n_{e,max}$ and the effective recombination coefficient α_{eff} by $n_{e,max} = [q_{max}/\alpha_{eff}]^{1/2}$.

Part 2)i)

Using the parameters below, determine q_{max} for each layer of the ionosphere:

First, find q_{max} using the provided equation... $q_{max} = n^2_{e,max} \cdot \alpha_{eff}$.
This provides the following values, in the right-hand most column:

Region	Peak Altitude	$n_{e,max}$ (cm^-3)	$\alpha_{eff}(cm^3/s)$	$q_{max}(pairs/m^3/s)$
E	110 km	$1.5 \cdot 10^5$	10^{-8}	225
F1	200 km	$2.5 \cdot 10^5$	$7 \cdot 10^{-9}$	437.5
F2	300 km	$5.0 \cdot 10^5$	$5 \cdot 10^{-9}$	1250

Note that F2 is not a "chapman" layer, because the production and loss balance is not well explained by solely using height as the independent variable.

Part 2)ii)

Explain why, when all other factors are kept constant, the ionization peak altitude (z_{max}) of a layer moves upward with increasing solar zenith angle χ.

The altitude at which peak ionization occurs increases with a more direct solar zenith angle because as the zenith angle increases (away from a direct angle), the altitude, h, effectively increases. An ray coming from a larger inclination will have to travel through "more ionosphere" before reaching the point in question, which reduces it's total capacity to ionize.

Exercise 3. (10) Spacecraft Charging in LEO:

What is the floating potential of an unbiased 6U CubeSat in low-Earth orbit at 400 km altitude, flying with its 20x30 cm face in the ram direction? Assume an electron temperature of 1000 K. What assumptions did you have to make?

Collect variables and consider the general floating potential equation. Assumptions include:
V_{fl} is based on a spherical spacecraft, v_{sc} calculated based on a circular orbit at 400km altitude. $v_{e,th}$ calculated as the mean of the magnitude of the electron velocity. The areas were calculated as flat surfaces.

$$A_i = .06\ m^2 \qquad A_e = 0.22\ m^2$$

$$v_{sc} = \sqrt{\frac{GM}{Re + h}} = 7672\ m/s \qquad q_e = 1.602 \cdot 10^{-19} C$$

$$k = 1.381 \cdot 10^{-23} kg\, m^2\, s^{-2}\, K^{-1} \qquad T_e = 1000\ K$$

$$v_{e,th} = \sqrt{8kT/\pi m} = 19646\ m/s$$

$$V_{fl} = \frac{kT_e}{q_e} \cdot ln\left(\frac{4v_{sc}A_i}{v_{e,th}A_e}\right) \rightarrow MATLAB$$

$$V_{fl} = -0.16\ Volts$$

Exercise 4, (10) Spacecraft Charging in GEO:

Consider a spacecraft in GEO with one surface in sunlight (normal to the sun's direction) and one surface in shadow. Use equation (19) and the relevant equations that follow to calculate the floating potential on each of these two surfaces. Assume a plasma density of $n_e = n_i = 0.5\ cm^{-3}$, and a plasma temperature $T_e = T_i = 5 \cdot 10^6 K$. You can ignore the SEE and backscatter currents. What is the potential difference between these two surfaces?
[Hint: you can assume the potential is negative in shadow and positive in sunlight. Note that you'll probably need to solve the equation for dQ/dt numerically.]

First note estimate for V_{fl} due to only electron and ion currents. Use this as a starting point for iteration... ($T_{eV}@5 \cdot 10^6 K = 431eV$)

$$V_{fl} \simeq -2.5 T_e \ \ (T\ in\ eV) = -2.5 \cdot k_B T_e \ \ (T\ in\ K) = -1077\ Volts$$

In the shadow, the floating potential is given by the following equation (where $V < 0$). Iterate this equation in matlab until equilibrium is reached.

$$V_{fl,shade} = \frac{kT_e}{q_e} ln \left[\sqrt{\frac{m_e}{m_i}} \left(1 - \frac{q_e * V_{fl}}{kT_i} \right) \right] = -1551\ Volts$$

In the sunlight, assume the potential is positive AND consider the photoelectric effect. Assume $A_e = A_i = A_n$. I_i and I_e and I_{ph} for $V > 0$:

$$I_e = -\tfrac{1}{4} q_e n_e v_{e,th} A_e \left(1 + \frac{q_e V}{k_B T_e} \right)$$

$$I_i = \tfrac{1}{4} q_e n_i v_{i,th} A_i \left(e^{-q_e V/(k_B * T_i)} \right)$$

$$I_{ph} = I_0 A_n cos(\theta) e^{-qV/(k_B T_{ph})} \qquad where \qquad k_B T_{ph} \simeq 2eV \rightarrow T_{ph} = 23209K$$

Solve in MATLAB by providing a range of potential floating point voltages and solving for $I_e + I_i + I_{ph}$ at each voltage. This provides data for $\sum I \ \ vs.\ V$. Interpolate for the voltage where $\sum I = 0$.

$$V_{fl,sun} = 12.66\ Volts$$

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Exercise 5, (10) Radio Wave Propagation:

Using the IRI profile you created in Problem 1,

i) What is the minimum radio frequency that can be used to communicate from ground to space?
ii) From which ionospheric region does this frequency reflect?
iii) From what altitude does a 2 MHz radio wave reflect?

i) To communicate from ground to space, we must first define the altitude at which space begins. Many organizations use a height of 100km, known as the Karman line, to define the start of space. In order to reach 100km, we must find the peak electron density between ground level and 100km. The IRI data indicates this occurs at 100 km, $n_{e,100km} = 71284cm^{-3}$. Then we use either of the equations below to find the minimum frequency that will pass through the defined electron density.

$$f_c[MHz] = 9 \cdot 10^{-3}\sqrt{n_{ec}[cm^{-3}]} = 0.009\sqrt{71284} = 2.402MHz$$

$$f = \frac{\omega_p = \sqrt{\frac{n_e q_e^2}{m_e \epsilon_0}}}{2\pi} = \frac{\sqrt{\frac{71284 \cdot 10^6 (1.6 \cdot 10^{-19})^2}{9.109 \cdot 10^{-31}(8.854 \cdot 10^{-12})}}}{2\pi} = 2.394\ MHz$$

However, if the intention is to communicate with spacecraft in conventional orbits, at 400km or above, we must find the peak electron density between the ground and 400km (or above). The IRI data indicates the peak electron density: $NmF2 = 559384.2cm^{-3}$ occurs at a height of 226 km.

$$f_c[MHz] = 9 \cdot 10^{-3}\sqrt{n_{ec}[cm^{-3}]} = 0.009\sqrt{559384.2} = 6.731MHz$$

$$f = \frac{\omega_p = \sqrt{\frac{n_e q_e^2}{m_e \epsilon_0}}}{2\pi} = \frac{\sqrt{\frac{5.5938 \cdot 10^{11} (1.6 \cdot 10^{-19})^2}{9.109 \cdot 10^{-31}(8.854 \cdot 10^{-12})}}}{2\pi} = 6.706\ MHz$$

ii) When considering the 100km definition of space and the corresponding $f_c = 2.4\ MHz$ frequency, we see that this frequency will be reflected if it travels any higher, because the electron density if still increasing (up to a height of 226km). This occurs in the E region.

However, when considering conventional orbit communication and the corresponding frequency $f_c = 6.73\ MHz$ and the height at which it occurs $h = 226km$, we see that this frequency will continue to travel further into space because it has already reached/passed the maximum electron density. This NmF2 point occurs in the F region, though there is no reflection of this frequency.

iii) Assuming A 2MHz radio wave will be reflected when the electron density first reaches $49.38cm^{-3}$...

$$(\frac{f_c[MHz]}{.009})^2 = n_e[cm^{-3}] = (\frac{2}{.009})^2 = 49.38cm^{-3}$$

The IRI data indicates this happens at an altitude of 97 km (interpolated).

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Exercise 6, (10) Radio Blackout:

During spacecraft re-entry, as demonstrated in Hidden Figures, the spacecraft temporarily loses the ability to communicate with the ground.

i) Describe in a few sentences why this happens.

ii) If the module normally communicates with the ground at 2282.5 MHz, what must the minimum electron density be in the plasma sheath region?

i) This happens because as the spaceecraft travels through the increasingly dense atmosphere, the craft compresses the atmosphere which leads to a large increase in temperature up to 10000 to 12000 Kelvin. This combination of pressure and temperature create an ionized plasma "cloud" around the craft with an electron density large enough to cause it's plasma frequency to be much higher than the communication frequencies we rely on (i.e. our communication frequencies are blocked by the ionized plasma with a large electron density).

ii)If the module normally communicates with the ground at 2282.5 MHz, the minimum electron density is given by the equation, where n_{ec} is in cm^{-3}:

$$f_c[MHz] = 0.009 \cdot \sqrt{n_{ec}[cm^{-3}]} \rightarrow n_{ec} = \left(\frac{f_c}{0.009}\right)^2 = \left(\frac{2282.5}{0.009}\right)^2$$

$$n_{ec} = 64.3 \cdot 10^9 \; cm^{-3}$$

Exercise 7, (10) GPS Delays:

i) Calculate the path length delay through the ionosphere for 10 TECU, at $L1 = 1575.42$ MHz and at $L2 = 1227.60$ MHz.

ii) Explain how you can use dual-frequency GPS to reduce the path length (and thus position) errors due to the ionosphere.

i) Find the path length delay, converting everything into SI units...

$$\Delta l_{iono,L1} = -\frac{40.3}{f^2} * TEC = -\frac{40.3}{(1575.42 * 10^6)^2} * 10 * 10^{16} = 1.623 \; meters$$

$$\Delta l_{iono,L2} = -\frac{40.3}{f^2} * TEC = -\frac{40.3}{(1227.60 * 10^6)^2} * 10 * 10^{16} = 2.674 \; meters$$

ii) Instead of using one, higher frequency signal that would have a shorter path length delay, dual-frequency GPS techniques can be used. By measuring the path delay for two frequencies emitted from the same source, you can indirectly measure/solve for the TEC of the GPS path. This informs what the error is, allowing for a position correction from ionospheric noise.

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Exercise 8 (10) D-region Signal Attenuation:

Calculate the attenuation (in dB) of a 10 MHz signal propagating vertically through your IRI ionosphere from Problem 1. Use your MSIS atmosphere profile from the previous Module to determine the collision frequency, and assume $|B_0| = 50,000$ nT.

Find the collision frequency, ν_{en} using the HW2 MSIS data. Assume $T_e = T_{neutral}$ because there are many collisions to "equilibrate" the particles in the D-region. MATLAB provides ν_{en} as a function of altitude.

$$\nu_{av}(e, N2) = 2.33 \cdot 10^{-17} n_{N_2} \cdot (1 - 1.21 \cdot 10^{-4} T_e) T_e$$
$$\nu_{av}(e, O2) = 1.82 \cdot 10^{-16} n_{O_2} \cdot (1 + 0.036 T_e^{1/2}) T_e^{1/2}$$
$$\nu_{en} = \nu_{av}(e, N2) + \nu_{av}(e, O2)$$

Consider the provided absorption equation, use the electron density from the IRI data.

$$\omega_c = \frac{q_e B_0}{m_e} = 8.79 \cdot 10^6 rad/s, \qquad \omega = 2\pi f = 2\pi 10 \cdot 10^6 = 62.83 \cdot 10^6 rad/s$$
$$\frac{dA}{dl}[dB/m] = 4.61 \cdot 10^{-5} \frac{n_e \nu_{cm}}{\nu_{em}^2 + (\omega \pm \omega_c)^2}$$

For maximum absorption, consider when the $\pm$ term in the above equation is acting as a minus. A "differential absorption" and "total absorption" plot can be created, as shown on the next page. Note almost all the attenuation occurs between 0 and 125 km. The total attenuation of a 10 Mhz signal from 0 to 1000 km is 1.183 dB.

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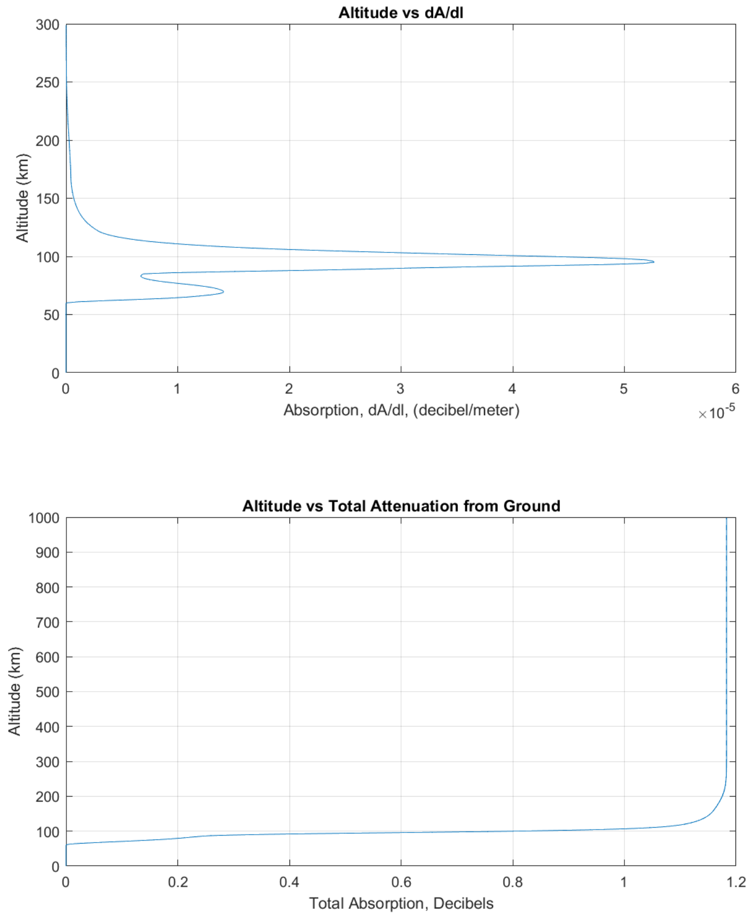

Figure 3: Part 8) Altitude vs. $\frac{dA}{dl}$ [decibels/meter]

Exercise 9 (10) ICON Mission:

The ICON Mission has four instruments: MIGHTI, IVM, EUV, and FUV (thank Berkeley for those creative acronyms). Do some research and pick your favorite instrument. In a couple of paragraphs, explain:
- What does this instrument do?
- What is the instrument SWAP (size, weight, and power)?
- In basic terms, how does it work?
- How does it contribute to the science goals of the ICON mission – in particular, what information about the ionosphere does it provide?

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From the following webpages:
https://icon.ssl.berkeley.edu/Observatory/Instruments/FUV
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6121705/

• The IVM (Ion Velocity Meter) instrument is derived from another instrument operating on the CINDI project on the C/NOFS satellite. This is to say that ion velocity measurements are common measurements for various missions. The instrument is self explanatory - it measures ion velocity by using a planar sensor/collector normal to the incoming plasma stream. It is capable of measuring Ion drift dynamic range, vertical ion drift accuracy, total ion concentration, and horizontal sample distance.
• The size is roughly 33cm tall, 20cm wide, and 14cm deep. It's mass is 4.25 kg and consumes a max of 2.5W of power.
• By facing an aperture in the direction of travel, incoming ions/plasma, travel through a grounded grid (which ensures internally applied potentials do not influence ion trajectory), then travel through additional grids of certain potential to filter ions by energy level. The signal produced by the collector plate is converted into information about the ion such as velocity or total current as a function of retarding voltage.
• ICON aimed to study changes in the ionosphere and interactions between earth and space weather. IVM contributed to this mission by providing important observations about particles in response to the push of high altitude winds and electric fields. More specifically, "The primary measurement goal for the IVM is to provide the meridional ion drift perpendicular to the magnetic meridian with an accuracy of $7.5m/s$ for all daytime conditions encountered by the spacecraft within 15 degrees of the magnetic equator".

MATLAB Code

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% ASEN 5335 HW3
% Ross Fischer
% 10/02/2023

clc, clear all, close all

%% Question 1
IRI = readtable('IRI_Model_Output.txt');
IRI.Properties.VariableNames = {'alt', 'Ne/cm-3', 'Ne/NmF2', 'Tn/K', 'Ti/K', 'Te/K', 'O+', 'N+
IRI.("Ne/cm-3")(IRI.("Ne/cm-3")<0) = 0; %set negative percentages to zero
IRI.("O+")(IRI.("O+")<0) = 0; %set negative percentages to zero
IRI.("N+")(IRI.("H+")<0) = 0;
IRI.("H+")(IRI.("H+")<0) = 0;
IRI.("O2+")(IRI.("O2+")<0) = 0;
IRI.("NO+")(IRI.("NO+")<0) = 0;
IRI.("He+")(IRI.("He+")<0) = 0;
IRI.("O+Density")=IRI.("Ne/cm-3").*IRI.("O+")/100;
IRI.("H+Density")=IRI.("Ne/cm-3").*IRI.("H+")/100;
IRI.("O2+Density")=IRI.("Ne/cm-3").*IRI.("O2+")/100;
IRI.("NO+Density")=IRI.("Ne/cm-3").*IRI.("NO+")/100;
IRI.("He+Density")=IRI.("Ne/cm-3").*IRI.("He+")/100;

figure; hold on
plot(IRI.("Ne/cm-3"), IRI.alt, 'k--')
plot(IRI.("O+Density"), IRI.alt)
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```
plot(IRI.("H+Density"), IRI.alt)
plot(IRI.("O2+Density"), IRI.alt)
plot(IRI.("NO+Density"), IRI.alt)
plot(IRI.("He+Density"), IRI.alt)
hold off
axis([0 6*10^5 0 1000])
xscale log
title('Altitude vs. Constituent # Density')
ylabel('Altitude (km)', 'Rotation',0)
xlabel('# Density (1/cm^3)')
legend('Total','O+', 'H+', 'O2+', 'NO+', 'He+')
grid on

%1)iii
IRI.TEC2 = cumtrapz(IRI.("alt")*1000,IRI.("Ne/cm-3")*1000000)/(10^16);
figure
plot(IRI.TEC2,IRI.alt)
title('Total Electron Current vs. Altitude')
ylabel('Altitude (km)', 'Rotation',0)
xlabel('TECU (1 TECU = 10^{16} electrons/m^2)')
grid on

%% question 3
me = 9.10938 * 10^(-31); %kg electron mass
mi = 2.58 * 10^(-26); %kg of ion
qe = 1.602176 * 10^(-19); %coulomb
k=1.38065*10^(-23); %kg m^2 k^-1 s^-2

Ai=20*30/10000; %m^2
Ae=.22; %m^2
vsc = 7672; %m/s
Te1 = 1000; % Kelvin

veth1 = sqrt(8*k*Te1/(pi()*me));

Vfl1 = k*Te1/qe * log(4*vsc*Ai/(veth1*Ai))

%% question 4 spacecraft charging in GEO

ne2 = 0.5*1000000; % ne = ni = m^-3
Te2 = 5000000; %Kelvin, T_e = T_i
veth2 = sqrt(8*k*Te2/(pi()*me));
vith2 = sqrt(8*k*Te2/(pi()*mi));
theta = 0; %incident angle to sun
I_J = 40 * 10^(-6); % amps/m^2 constant current density
kTph = 2; %photon temperature, eV
Tph = 23209; %photon temp, kelvin

Vfl_shade(1) = -2.5*Te2/11606 %initial Voltage
% Iph = .000040 * A
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   % Vfl_sun(i) = (k*Te2/qe) * ((.25 * qe * ne2 * vith2 * exp(-qe*Vfl_sun(i-1)/(k*Tph)) + I_J

for i=2:10 %arbitrary number of iterations to solve for V
   Vfl_shade(i) = (k*Te2/qe)*log(sqrt(me/mi)*(1-(qe*Vfl_shade(i-1)/(k*Te2))));
end

Vfl_sun = 5:.05:20; %initial guess array for sunlight voltage
j=1;
for i=Vfl_sun
    sumI(j) = (-0.25 * qe * ne2 * veth2 * (1 + qe*i/(k*Te2)) + 0.25 * qe * ne2 * vith2 * exp(-
    j=j+1;
end
clear j; clear i
interp1(sumI,Vfl_sun,0)

%% question 8
for i=1 %A matrix from HW2 MSIS data
A = [a lot of data]

% Find nu_en (collision freq)
nN2 = A(:,3)*1000000; % #/m^3
nO2 = A(:,4)*1000000; % #/m^3
TempNeu = A(:,6); % Kelvin
nuEN = (2.33 * 10^(-17) * nN2 .* (1-1.21*10^(-4)*TempNeu).*TempNeu) + (1.82*10^(-16)*nO2.*(1+0

B = 0.000050000 % Tesla
omegaGyro = qe * B / me; % rad/s
omega = 2*pi*10000000; % rad/s
alt = A(:,1)*1000; % meter
ne8 = interp1(IRI.("alt")*1000, IRI.("Ne/cm-3"), alt, 'pchip')*1000000; % electron #/m^3
% dAdl = 4.61 * 10^(-5) * (ne8.*nuEN)./(nuEN.^2 + (omega + omegaGyro)^2); %minimum absorption?
dAdl2 = 4.61 * 10^(-5) * (ne8.*nuEN)./(nuEN.^2 + (omega - omegaGyro)^2); %maximum absorption
attenuation = cumtrapz(alt,dAdl2)

figure
tiledlayout(2,1);
tile1=nexttile;
plot(dAdl2(1:300),alt(1:300)/1000)
title('Altitude vs dA/dl')
xlabel('Absorption, dA/dl, (decibel/meter)')
ylabel('Altitude (km)')
grid on
tile2=nexttile;
plot(attenuation,alt/1000)
title('Altitude vs Total Attenuation from Ground')
xlabel('Total Absorption, Decibels')
ylabel('Altitude (km)')
grid on
```